SCHOLAR Study Guide

Higher Physics
Unit 1: Our dynamic universe

Authored by:

Ian Holton (previously Marr College)

Reviewed by:

Grant McAllister (Bell Baxter High School)

Previously authored by:

Douglas Gavin

John McCabe

Andrew Tookey

Campbell White

Heriot-Watt University

Edinburgh EH14 4AS, United Kingdom.

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Topic 1

Motion: equations and graphs

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Learning objective

By the end of this topic you should be able to:

- state the difference between scalars and vectors;
- add vectors;
- find the rectangular components of a vector;
- derive the equations of motion;
- solve problems using the equations of motion;
- draw acceleration-time, displacement-time graphs and velocity-time graphs, and use them to deduce information about the motion of an object;
- calculate the acceleration of an object falling near the Earth's surface from data on a velocity-time graph;
- investigate how the angle of a frictionless slope affects the acceleration of an object.

We will begin this unit by thinking about how different quantities can be classified as scalars or as vectors. We will then examine vector quantities in detail learning how to combine and analyse them. We will then start to examine objects that are moving.

- If you drop a book out of the window, how long does it take to reach the ground? How fast is it travelling when it hits the ground? In this topic we will study the vector quantities displacement, velocity and acceleration so that we can answer questions on the motion of objects. There are many practical examples that we will be able to analyse, such as the motion of cars as they accelerate or slow down.
- Throughout the topic we will be concentrating on objects moving with constant acceleration. We will also use graphs of acceleration, velocity and displacement plotted against time to give a graphical representation of the motion of an object.
- We will derive four equations, known as the kinematic relationships or the equations of motion. We can use these equations to solve problems involving motion with constant acceleration and examine how these relationships can be verified by experiments.
- We will look at the type of equipment that can be used in the laboratory to measure displacement, time and velocity and hence calculate acceleration.
- Once you have learned about acceleration and the equations of motion, we will find out how to analyse the motion of an object using graphs.
- Finally we will look at two particular cases of accelerating objects: an object in freefall and an object accelerating down a slope.

1.1 Vectors

Vectors play an important role in Physics. You will be using vectors to describe a large number of different physical quantities throughout this course, so this subtopic provides you with all the skills you need to use vectors. We will deal with two issues: how to add vectors, and how to find components of a vector. You will find that these two tasks appear time and time again as you progress through the course. If you pick up a good understanding of vectors now, other Physics Topics will be made a lot simpler.

1.1.1 Distance and displacement

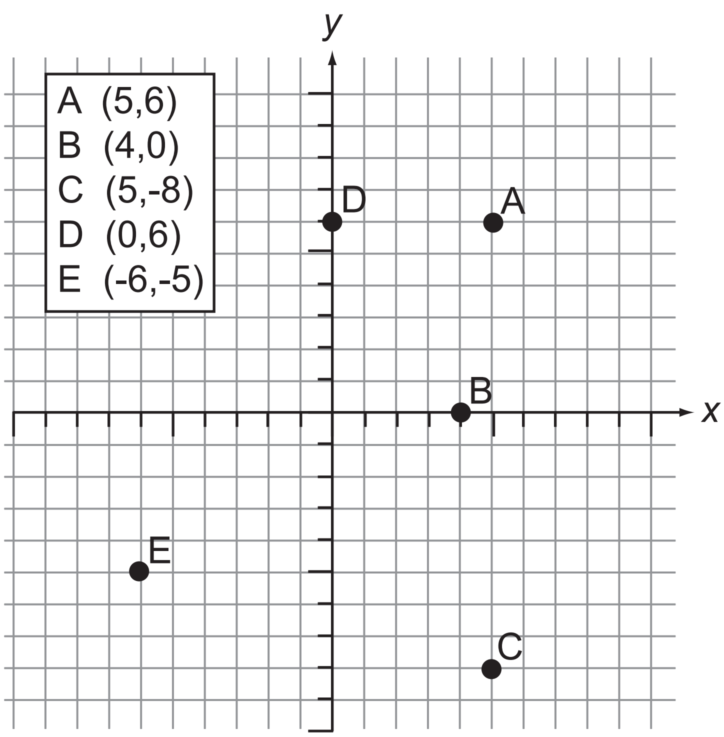
Consider these two situations. On Monday morning, you leave your house and walk directly to school, 500 m from your home. On Tuesday morning, instead of walking directly to school, you take a turn off the direct route to your friend's house. From there you walk to the newsagent to buy a magazine, and from there you walk to school.

Now, in both of these cases, you have ended up a distance of 500 m from where you started, but clearly on Tuesday you have walked a lot further to get there. In Physics we distinguish between your displacement and the distance you have travelled.

The **displacement** of an object from a particular point is defined as the distance in a specified direction between that point and the object. When we are talking about displacement, we are not concerned with the distance travelled by the object, only its direct distance from the starting point. So even if your route to school on Tuesday covered 800 m, your final displacement from home is still 500 m.

Displacement is an example of a **vector** quantity. A vector is a quantity that has direction, as well as magnitude. Distance is called a **scalar** quantity; one which has magnitude but no direction. To illustrate the difference between vectors and scalars, consider Figure 1.1.

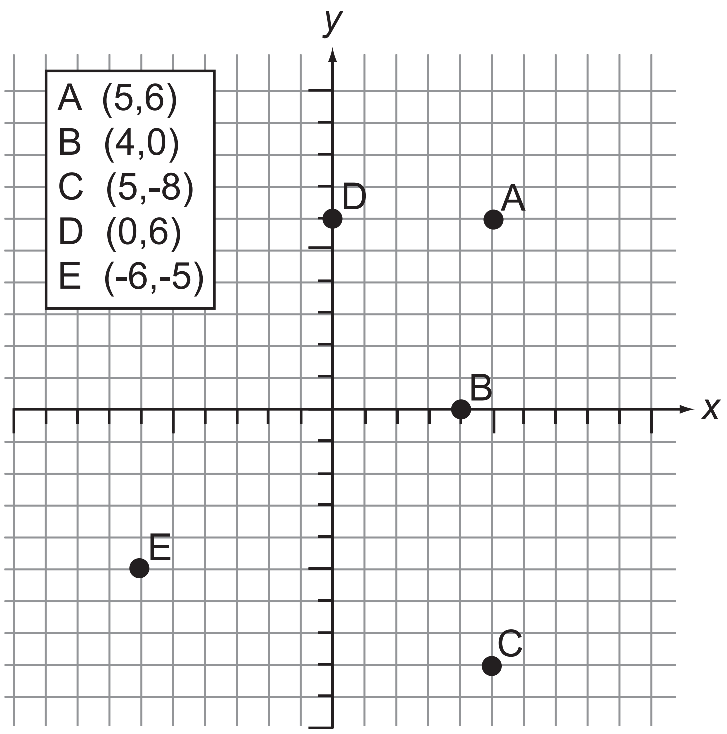
Figure 1.1: Points at different displacements from the origin



The coordinates of the points A, B, C, D and E are as shown. Let us now consider a direction, such as the x-direction. Although they are at different distances from the origin, points A and C have the same displacement in the x-direction (given by their x-coordinate). Point D has zero displacement in the x-direction, whilst point E has a negative x-displacement.

Distance and displacement

Answer the following questions which refer to the diagram below.



Q1: Which point has the same displacement in the y-direction as point A?

. . . .

Q2: Which point has a displacement of zero in the y-direction?

. . . .

Q3: Which point has a negative x-displacement *and* a negative y-displacement?

. . . .

Q4: Which point is at the same distance from the origin as point A, but with different x- and y-displacements?

1.1.2 Other vector and scalar quantities

We have seen that we can make a distinction between distance and displacement, since one is a scalar quantity and the other is a vector quantity. There are also scalar and vector quantities associated with the rate at which an object is moving.

Speed is a scalar quantity. If we say an object has a speed of 10 m s^{-1}, we are not specifying a direction. The **velocity** of an object is its speed in a given direction, so velocity is a vector. We would say that an object is moving with a velocity of 10 m s^{-1} in the x-direction, or a velocity of 10 m s^{-1} in a northerly direction, or any other direction.

Just as with displacement, it is important to specify the direction of velocity. Suppose we throw a ball vertically upwards into the air. If we specify upwards as the positive direction of velocity, then the velocity of the ball will start with a large value, and decrease as the ball travels upwards. At the highest point of its motion, the velocity is zero for a split second before the ball starts moving downwards. When it is moving back towards the Earth, the velocity of the ball is negative, because we have defined *upwards* as the positive direction.

We have described the difference between the scalar quantities distance and speed and the vector quantities displacement and velocity. Which other physical quantities are scalar quantities, and which are vector quantities?

Acceleration is the rate of change of velocity, and so it is a vector quantity. Force is also a vector quantity. Along with displacement and velocity, acceleration and force are the most common vector quantities we will be using in this course.

Amongst the other scalar quantities you will have already met in Physics are mass and temperature. There is no direction associated with the mass or temperature of an object. Scalar quantities are combined using the normal rules of mathematics. So if you have a 5.0 kg mass, and you add a 3.5 kg mass to it, the combined mass is 5.0 + 3.5 = 8.5 kg. As we will see in the next section, it is not always so straightforward to combine two or more vectors.

Examples are given below.

Vectors	**Scalars**
displacement	distance
velocity	speed
acceleration	time
force	mass
impulse	energy

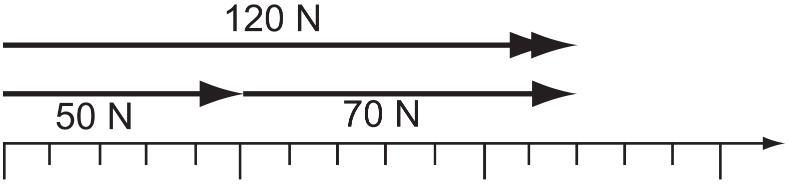
1.1.3 Combining vectors

What happens when we add two (or more) vectors together? We will start by looking at the simplest case, which is two vectors acting in the same dimension.

For example, if two men are trying to push-start a car, one may be applying a force of 50 N, the other may be applying a force of 70 N. If the two forces are acting in the same direction, the resultant force is just the sum of the two, 50 + 70 = 120 N.

We can obtain the same result if we use a scale drawing, as shown in Figure 1.2. Draw the two vectors "nose-to-tail", in either order, and the resultant is equal to the total length of the two vectors, 120 N.

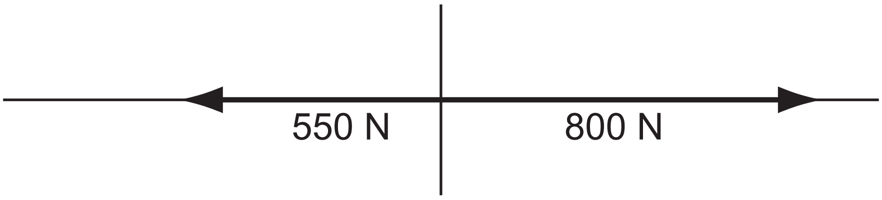
Figure 1.2: Collinear vectors acting in the same direction



Two vectors acting in the same direction are called collinear vectors.

What about two vectors acting in opposite directions, like the opposing forces in a tug-of-war contest? Suppose one tug-of-war team pulls to the right with a force of 800 N, while the other team pulls to the left with a force of 550 N, as shown in Figure 1.3.

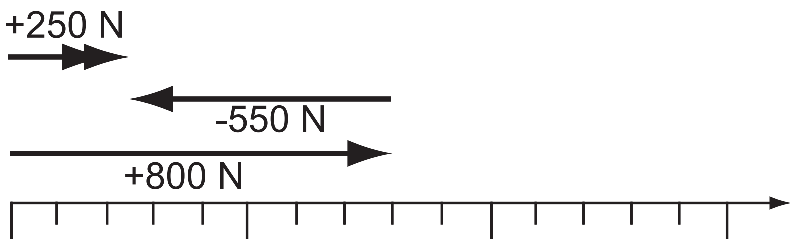
Figure 1.3: Two forces acting in opposite directions



The resultant force can be found by adding the two vectors, but remember that a vector has direction as well as magnitude. Common sense tells us that the tug-of-war team pulling with the greatest force will win the contest. Acting to the right, we have forces of +800 N and -550 N, so the total force acting to the right is 800 -550 = +250 N. This process is called finding the vector sum of the two vectors.

Again, we can use a nose-to-tail vector diagram, as in Figure 1.4. The resultant force is 250 N to the right.

Figure 1.4: Collinear vectors acting in opposite directions



We can combine as many collinear vectors as we like by finding their vector sum.

Adding collinear vectors Go online

At this stage there is an online activity. If however you do not have access to the internet you may try the questions which follow.

Q5: What is the resultant of adding vectors +20, +30 and -10?

. . . .

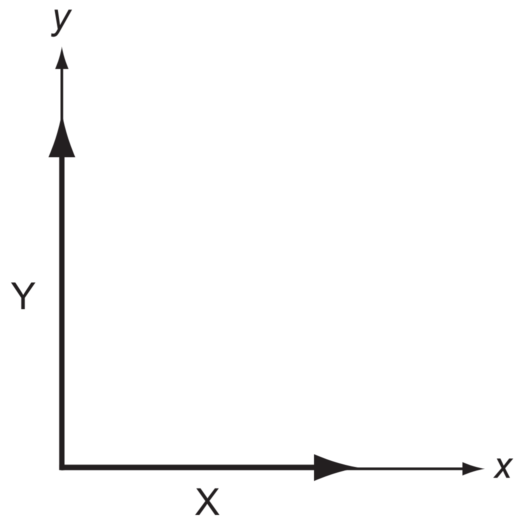
Q6: What is the resultant of adding vectors -10, -20, -30 and +20?

. . . .

Q7: What fourth vector should be added to -20, +10 and +50 to give a resultant of zero?

The next case to look at is the addition of two vectors which act at right angles to each other, sometimes called rectangular, orthogonal or perpendicular vectors. We can consider the general case of a vector X acting in the positive x-direction, and a vector Y acting in the positive y-direction. Figure 1.5 shows two perpendicular vectors.

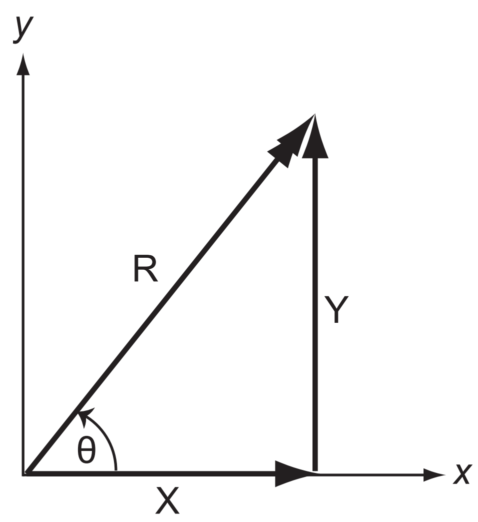
Figure 1.5: Perpendicular vectors



We could compare this situation to that of a man walking a certain distance to the east, who then turns and walks a further distance northwards. Again, common sense tells us his total displacement will be somewhere in a north-easterly direction, which we can find by the vector addition of the two east and north displacement vectors.

If we look at the nose-to-tail vector diagram in this case (Figure 1.6), we can see that the resultant is the hypotenuse of a right-angled triangle whose other sides are the two vectors which we are adding. So whenever we are combining two perpendicular vectors, the resultant is vector *R*, where the magnitude of *R* is $|R|$ which is given by $|R| = \sqrt{X^2 + Y^2}$. The direction of *R* is given by the angle θ, where $\theta = \tan^{-1}\left({}^{Y}\!/_{X}\right)$. So we would say that the resultant is a vector of magnitude *R*, acting at an angle θ to the *x*-axis.

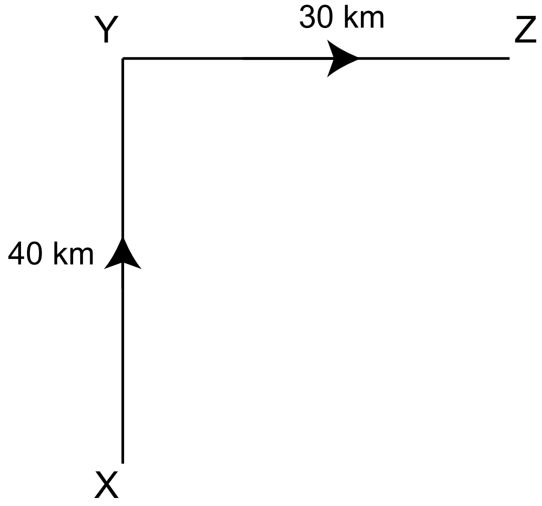
Figure 1.6: Resultant of two perpendicular vectors



Crossing the river Go online

At this stage there is an online activity. If however you do not have access to the internet you may try the questions which follow.

Q8: A car travels from X to Y and then Y to Z as shown.

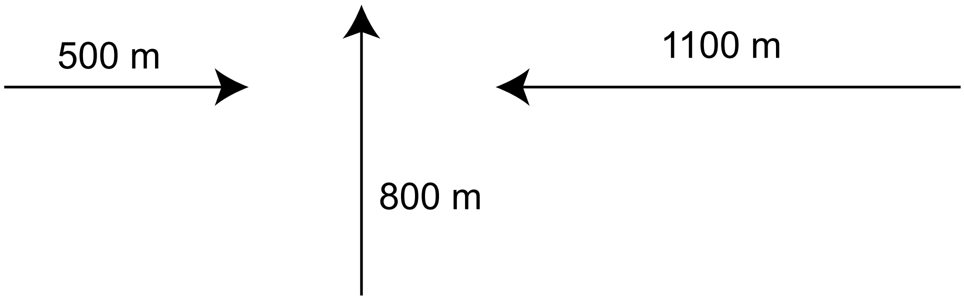


car journey

What is the magnitude of the displacement of the car?

. . . .

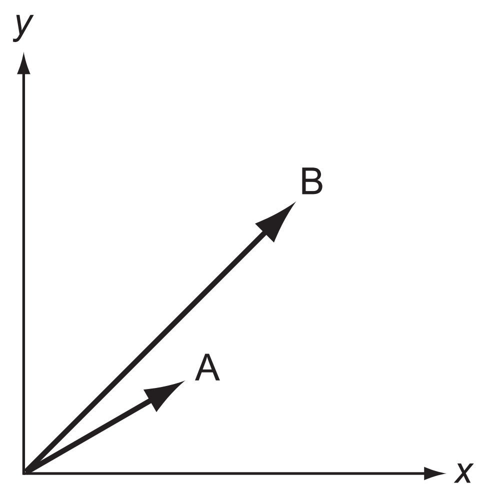
Q9: A competitor completes the following sequence of displacements in 10 minutes during part of an orienteering event.



What is the magnitude of the competitor's displacement?

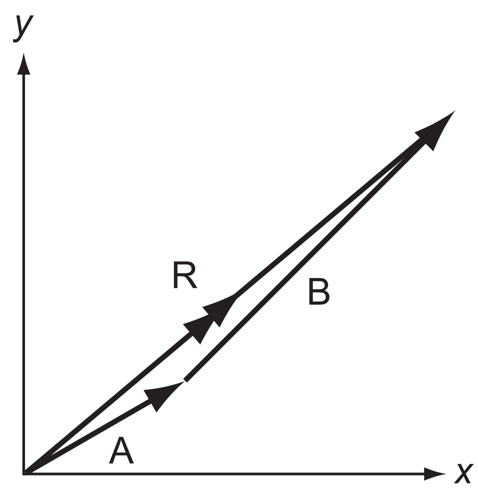
The general case of two vectors acting in different directions can be solved by using a scale drawing. As an example, let's consider a force *A* of magnitude 20 N, acting at 30° to the *x*-axis, and a force *B* of magnitude 40 N acting at 45° to the *x*-axis, where both forces act in the direction away from the origin. This set-up is shown in Figure 1.7

Figure 1.7: Two forces



An accurate scale drawing allows us to determine the magnitude and direction of the resultant force.

Figure 1.8: Scale drawing to determine the resultant of two vectors



In this case the scale drawing shows us that the magnitude of the resultant *R* is 60 N, and the direction of *R* (measured with a protractor) is 40° to the *x*-axis.

Again, we can find the resultant of any number of vectors by drawing them in scale, nose-to-tail.

Addition of vectors Go online

At this stage there is an online activity. If however you do not have access to the internet you may try the questions which follow.

Q10: A helicopter flies 20 km on a bearing of 180 (due South). It then turns on to a bearing of 140 (50° South of East) and travels a further 30 km.

By scale drawing find the resultant displacement of the helicopter.

. . . .

Q11: Competitors are racing remote control cars. The cars have to be driven over a precise route between checkpoints. Each car is to travel from checkpoint A to checkpoint B by following these instructions: "Drive 150 m due North, then drive 250 m on a bearing of 60° East of North (060)." By scale drawing, find the displacement of checkpoint B from checkpoint A.

Quiz: Adding vectors Go online

Q12: Two forces are applied to an object to slide it along the floor. One force is 75 N, the other is 40 N. If the two forces act in the same direction, what is the magnitude of the total force acting on the object?

a) 0.53 N
b) 1.875 N
c) 35 N
d) 85 N
e) 115 N

. . . .

Q13: What is the resultant force when the two following forces are applied to an object: a 25 N force acting to the north, and a 55 N force acting to the south?

a) 30 N acting northwards
b) 30 N acting southwards
c) 80 N acting northwards
d) 80 N acting southwards
e) 1375 N acting northwards

. . . .

Q14: Two perpendicular forces act on an object: a 120 N force acting in the positive x-direction, and a 70 N force acting in the positive y-direction. What is the magnitude of the resultant force acting on the object?

a) 9.5 N
b) 14 N
c) 90 N
d) 139 N
e) 190 N

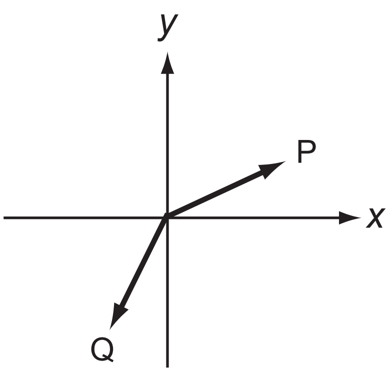
. . . .

Q15: Following on from the previous question, what is the angle between the resultant force and the x-axis?

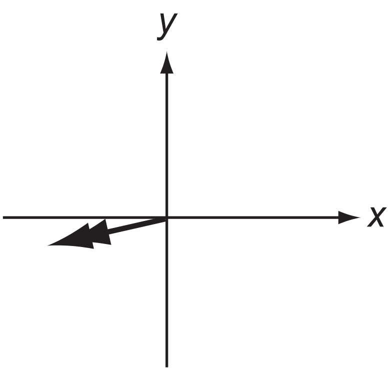
a) 1.7°
b) 30°
c) 36°
d) 54°
e) 60°

....

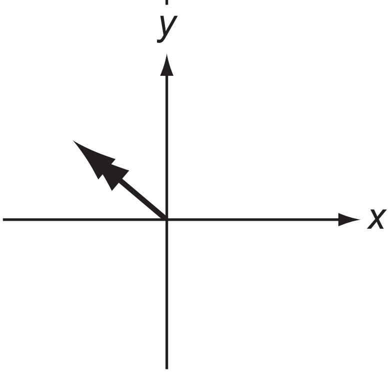
Q16: Consider the two vectors *P* and *Q* shown in the following diagram.



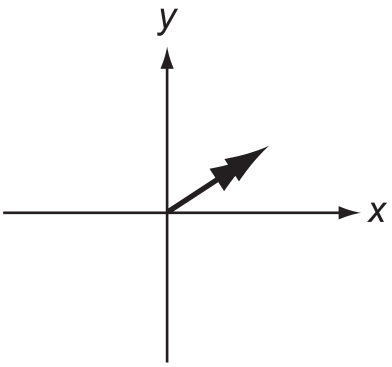
Which of the following could represent the resultant *R* of the two vectors *P* and *Q*?



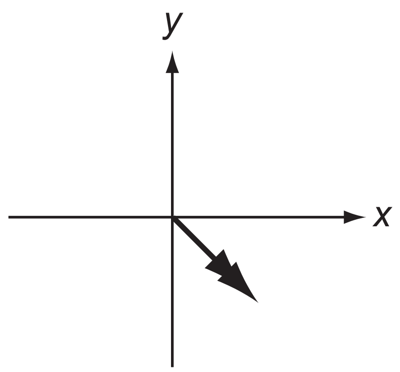
a)



b)



c)

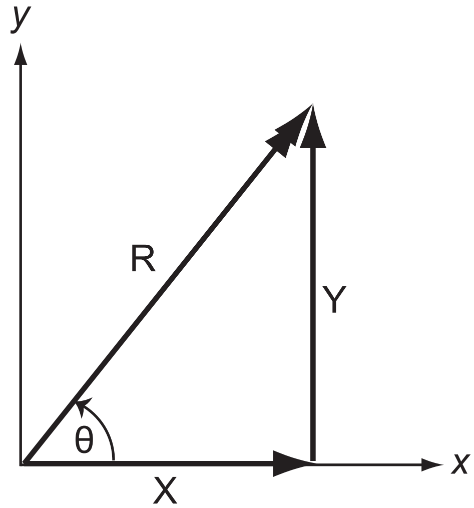


d)

1.1.4 Components of a vector

Looking back to the previous section, we saw that the resultant of two perpendicular vectors could be found using the laws of right-angled triangles. It is often useful for us to do the opposite process, and work out the perpendicular **components of a vector**. Let's look at the two vectors *X* and *Y* and their resultant *R*, shown again in Figure 1.9.

Figure 1.9: Perpendicular components of a vector



If we know the values of *R* and θ, we can work out the values of *X* and *Y* using the laws of right-angled triangles:

$$\sin\theta = \frac{Y}{R}$$
$$\therefore Y = R\sin\theta$$

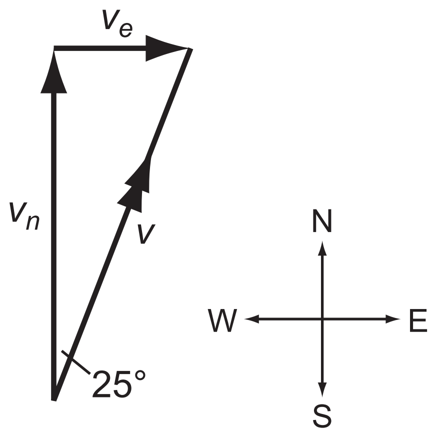
$$\cos\theta = \frac{X}{R}$$
$$\therefore X = R\cos\theta$$

We will meet many situations in Physics where we use the perpendicular components of a vector, so it is important that you are able to carry out this process.

Example

A car is travelling at 20 m s^{-1}. A compass on the dashboard tells the driver she is travelling in a direction 25° east of magnetic north. Find the component of the car's velocity

1. in a northerly direction;
2. in an easterly direction.



1. Referring to the diagram, the component v_n in the northerly direction is

$$v_n = v \times \cos 25$$
$$\therefore v_n = 20 \times 0.906$$
$$\therefore v_n = 18 \text{ m s}^{-1}$$

   The component in the northerly direction is 18 m s^{-1}

2. The component v_e in the easterly direction is

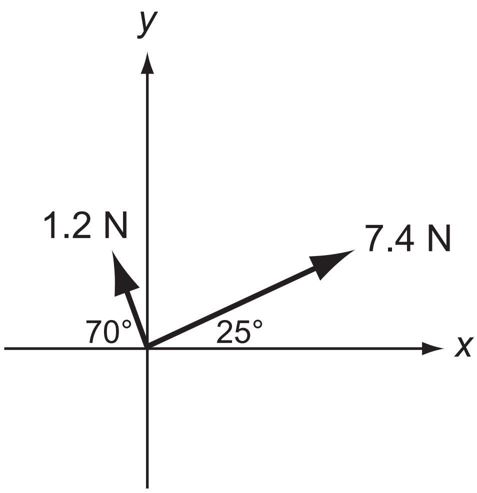
$$v_e = v \times \sin 25$$
$$\therefore v_e = 20 \times 0.423$$
$$\therefore v_e = 8.5 \text{ m s}^{-1}$$

   The component in the easterly direction is 8.5 m s^{-1}

One final point should be noted about the components of a vector. If we are adding two or more vectors together, we can use the components of each vector. If we find the x- and y-components, say, of each vector, then these components can be easily combined as they are collinear. Adding all the x-components together gives us the x-component of the resultant vector, and adding all the y-components together gives us its y-component. This method is often easier to use than making an accurate scale drawing.

Examples

1. The two forces shown in the diagram act on an object placed at the origin. By finding the rectangular components of the two forces, calculate the magnitude and direction of the resultant force acting on the object.



The y-components of the two forces both act in the positive direction, so the y-component R_y of the resultant is

$$R_y = (7.4 \times \sin 25) + (1.2 \times \sin 70)$$
$$\therefore R_y = 4.255\,\text{N}$$

The x-components of the two forces act in opposite directions, so the x-component R_x of the resultant is

$$R_x = (7.4 \times \cos 25) - (1.2 \times \cos 70)$$
$$\therefore R_x = 6.296\,\text{N}$$

The magnitude R of the resultant is

$$R = \sqrt{{R_y}^2 + {R_x}^2}$$
$$\therefore R = \sqrt{4.255^2 + 6.296^2}$$
$$\therefore R = 7.6\,\text{N}$$

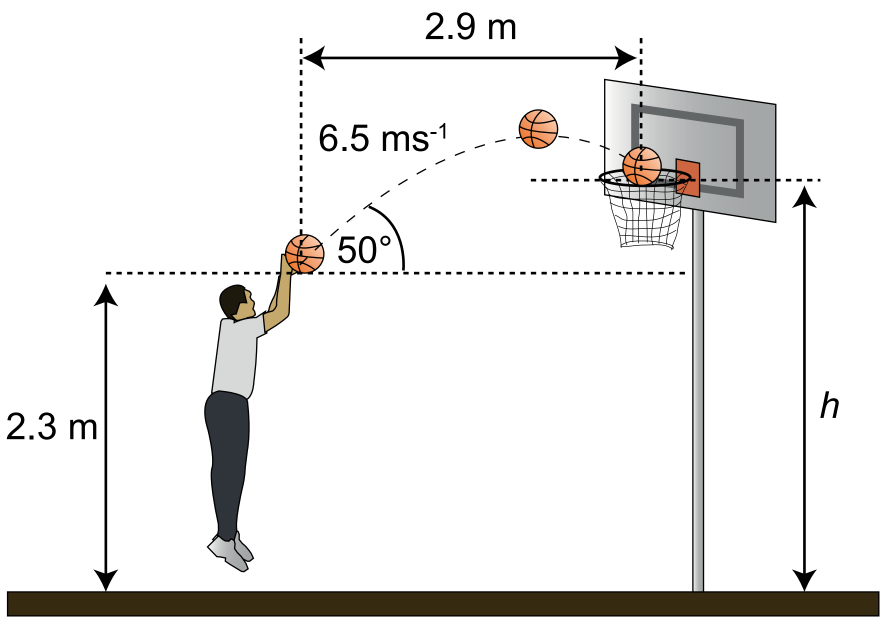
Both R_x and R_y act in a positive direction, so Racts in the (+x,+y) direction. The angle θ between R and the x-axis is

$$\theta = \tan^{-1}\left(\frac{4.255}{6.296}\right)$$
$$\therefore \theta = 34^\circ$$

The magnitude of the resultant force is 7.6 N and its direction is 34° from the x-axis.

....

2. A basketball player throws a ball with an initial velocity of 6.5 m s^{-1} at an angle of 50° to the horizontal.



Calculate:

1. the horizontal component of the initial velocity of the ball;
2. the vertical component of the initial velocity of the ball.

1. $u_h = 6.5 \cos 50° = 4.2$ m s^{-1}
   The horizontal component of the ball is 4.2 m s^{-1}
2. $u_v = 6.5 \sin 50° = 5.0$ m s^{-1}
   The vertical component of the ball is 5.0 m s^{-1}

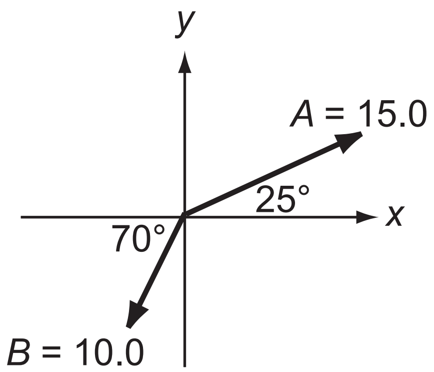
Quiz: Components of a vector Go online

Q17: A marksman fires his gun. The bullet leaves the gun with speed 320 m s^{-1} at an angle of elevation 40°. What is the horizontal component of the bullet's velocity as it leaves the gun?

a) 8.0 m s^{-1}
b) 206 m s^{-1}
c) 245 m s^{-1}
d) 268 m s^{-1}
e) 418 m s^{-1}

....

Q18: Consider the two vectors *A* and *B* shown in the diagram.



By considering the components of each vector, what is the y-component of the resultant of these two vectors?

a) -10.2
b) -3.06
c) +3.06
d) +10.2
e) +15.7

. . . .

Q19: What is the x-component of the resultant of the vectors A and B shown in the previous question?

a) -10.2
b) -3.06
c) +3.06
d) +10.2
e) +15.7

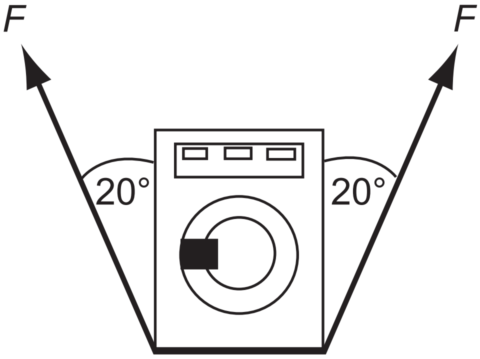
. . . .

Q20: A woman is dragging a suitcase along the floor in an airport. The strap of the suitcase makes an angle of 30° with the horizontal. If the woman is exerting a force of 96 N along the strap, what is the horizontal force being applied to the suitcase?

a) 24 N
b) 48 N
c) 62 N
d) 83 N
e) 96 N

. . . .

Q21: To carry a new washing machine into a house, two workmen place the machine on a harness. They then lift the harness by a rope attached either side. The ropes make an angle of 20° to the vertical, as shown in the diagram.



If each workman applies a force F = 340 N, what is the total *vertical* force applied to the washing machine?

a) 82 N
b) 230 N
c) 250 N
d) 320 N
e) 640 N

1.2 Acceleration

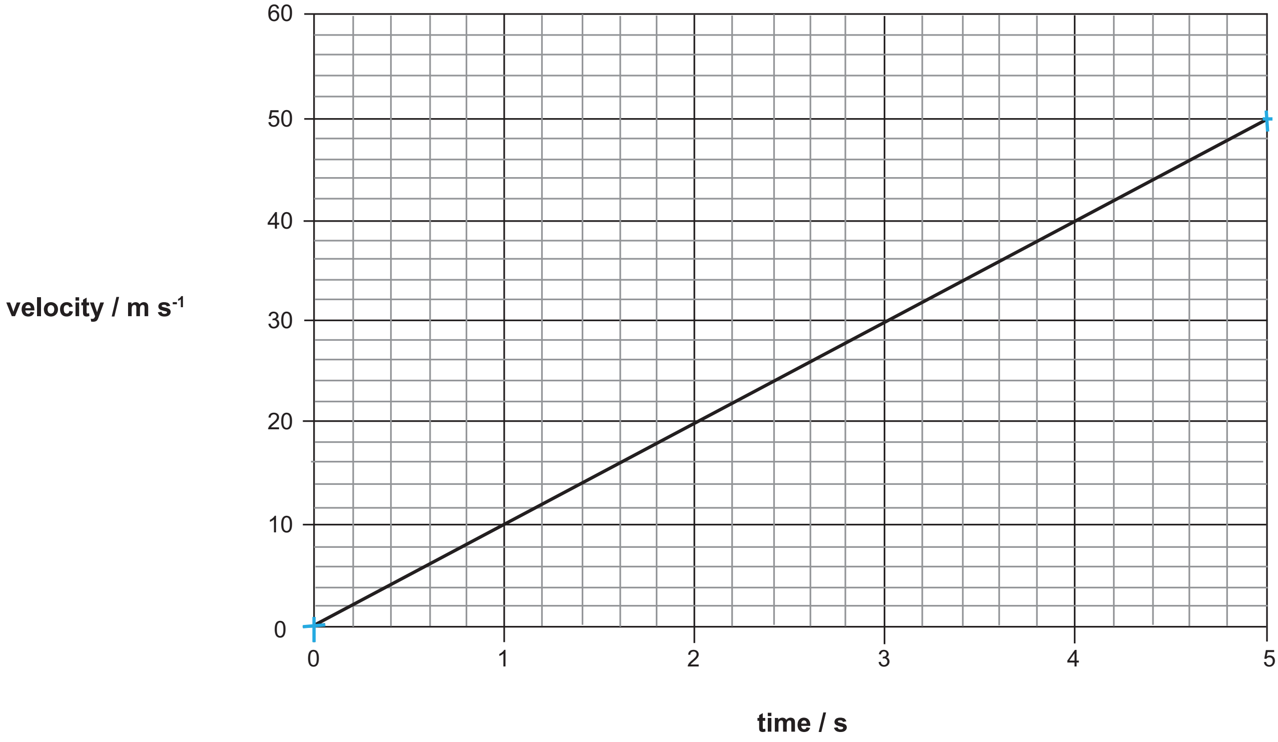
Acceleration (a) is defined as the change in velocity per unit time. This can also be expressed as the rate of change of velocity. Like velocity, acceleration is a vector quantity.

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$
$$a = \frac{\Delta v}{t} \tag{1.1}$$

The units of acceleration are m s^{-2}. We can think of this as "m s^{-1} per second". If an object has an acceleration of 10 m s^{-2}, then its velocity increases by 10 m s^{-1} every second. If its acceleration is -10 m s^{-2}, then its velocity is decreasing by 10 m s^{-1} every second. It is worth noting that an object can have a negative acceleration but a positive velocity, or vice versa.

The velocity of a moving object is often presented in a graph of velocity against time.

Consider an object that accelerates from rest to 50 m s^{-1} in a time of 5 seconds.
This motion is displayed in the following velocity-time graph.



The acceleration of the object could be calculated as follows.

$$a = \frac{\Delta v}{t}$$

$$a = \frac{v - u}{t}$$

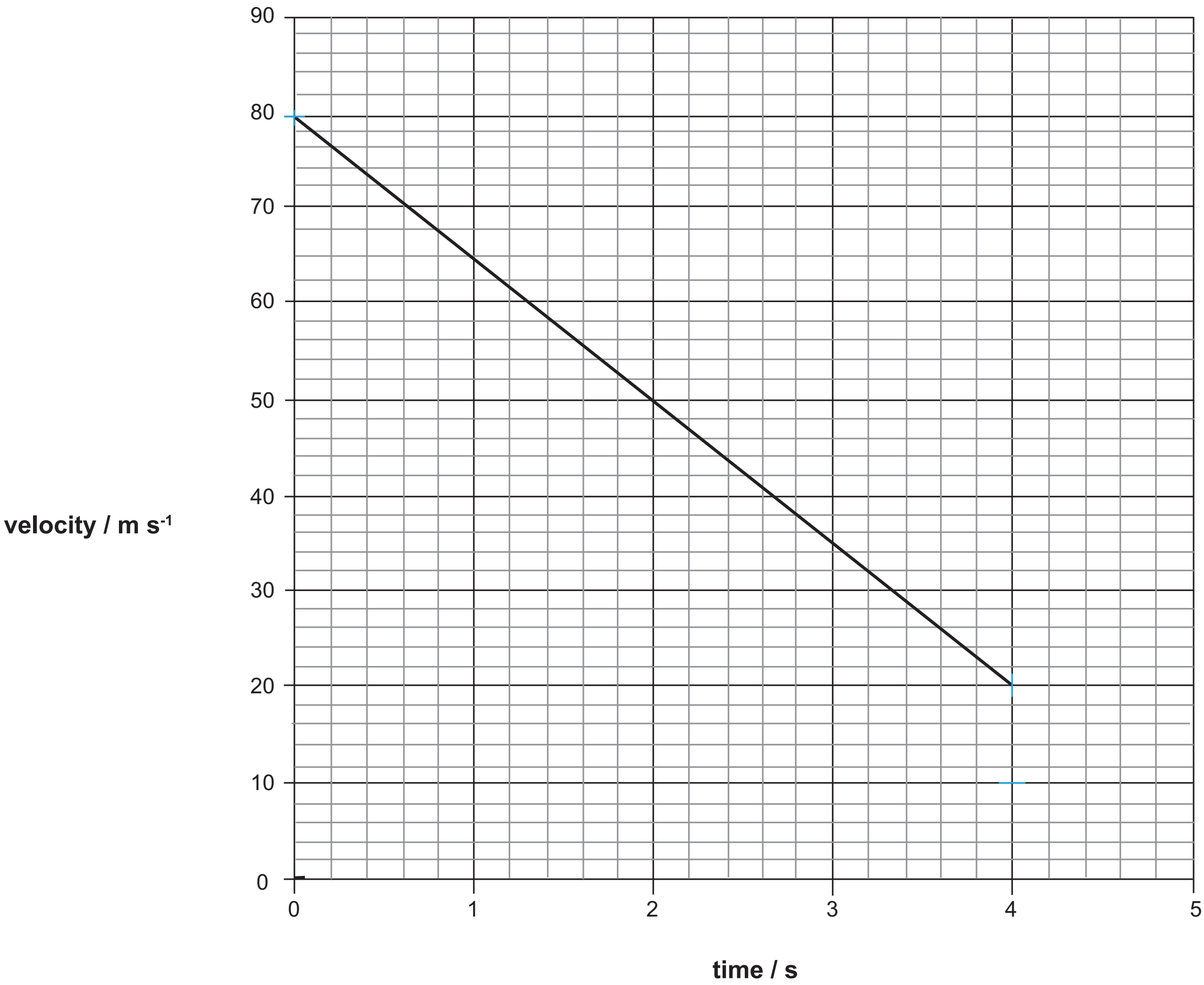
$$a = \frac{50 - 0}{t}$$

$$a = 10 \text{ m s}^{-2}$$

Looking at the graph is should be noticed that the velocity increases by 10 m s-1 in every one second.

This means that the acceleration is uniform and no matter which two points are selected on the graph the calculated acceleration will be the same.

The following velocity time graph shows the velocity decreasing with time.



The acceleration of the object could be calculated as follows.

$$a = \frac{\Delta v}{t}$$
$$a = \frac{v - u}{t}$$
$$a = \frac{20 - 80}{4}$$
$$a = -15 \text{m s}^{-2}$$

Looking at the graph is should be noticed that the velocity decreases by 15 m s-1 in every one second.

This means that the acceleration is uniform and no matter which two points are selected on the graph the calculated acceleration will be the same.

1.3 Kinematic relationships

Acceleration is the rate of change of velocity. Remember that we are considering only objects which move with a uniform acceleration. If we have an object whose velocity changes from an initial value *u* to a final value *v* in time *t*, then the acceleration *a* (= rate of change of velocity) is given by

$$a = \frac{\Delta v}{t}$$
$$a = \frac{v - u}{t}$$

We can rearrange this equation:

$$a = \frac{v - u}{t}$$
$$\therefore\ at = v - u$$
$$\therefore v = u + at$$

(1.2)

This is an important equation for an object moving with constant acceleration, and you will need to be able to apply it.

If the object's velocity changes from *u* to *v* in time *t*, then the average velocity is

$v_{av} = 1/2(u + v)$

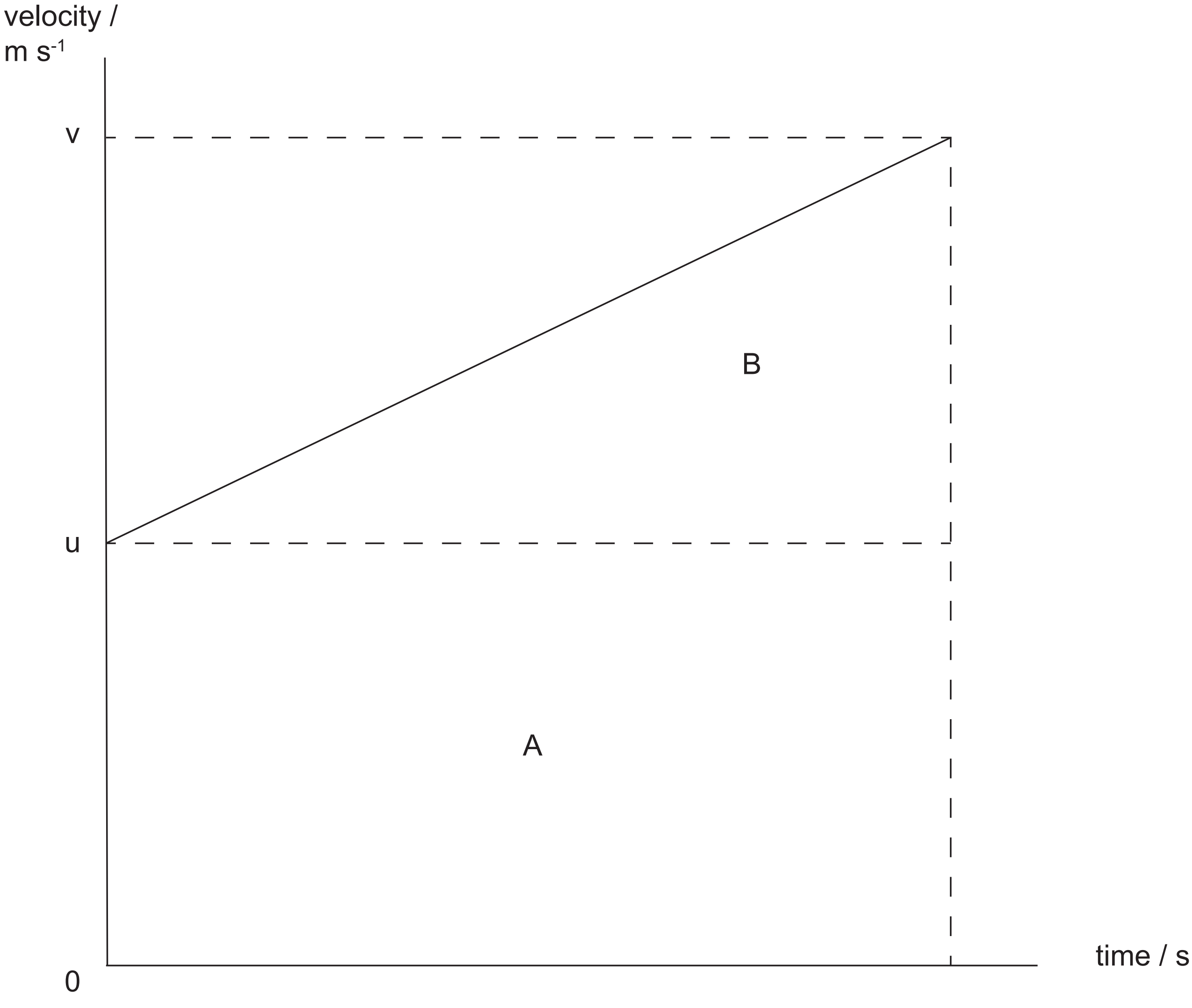
The displacement *s* of the object in time *t* is equal to the average velocity multiplied by *t*. That is to say,

s = area under velocity-time graph
s = area A + area B
s = (length x breadth) + 1/2 (base x height)
s = (t x u) + 1/2 (t x (v-u))
s = ut + 1/2 vt - 1/2 ut
s= 1/2 ut + 1/2 vts = 1/2(u + v)t

(1.3)

An alternative approach to deriving Equation 1.3 is to consider a velocity-time graph.

The following velocity-time graph is for an object accelerating uniformly from u to v in time t.



The displacement s during this acceleration is represented by the area under the velocity-time graph.

Since we know that $v = u + at$ we can substitute for v in the equation for s:

s = area under velocity-time graph
s = area A + area B
s = (length x breadth) + $1/2$ (base x height)
s = (t x u) + $1/2$ (t x (v-u))
but we know

$$a = \frac{(v - u)}{t}$$

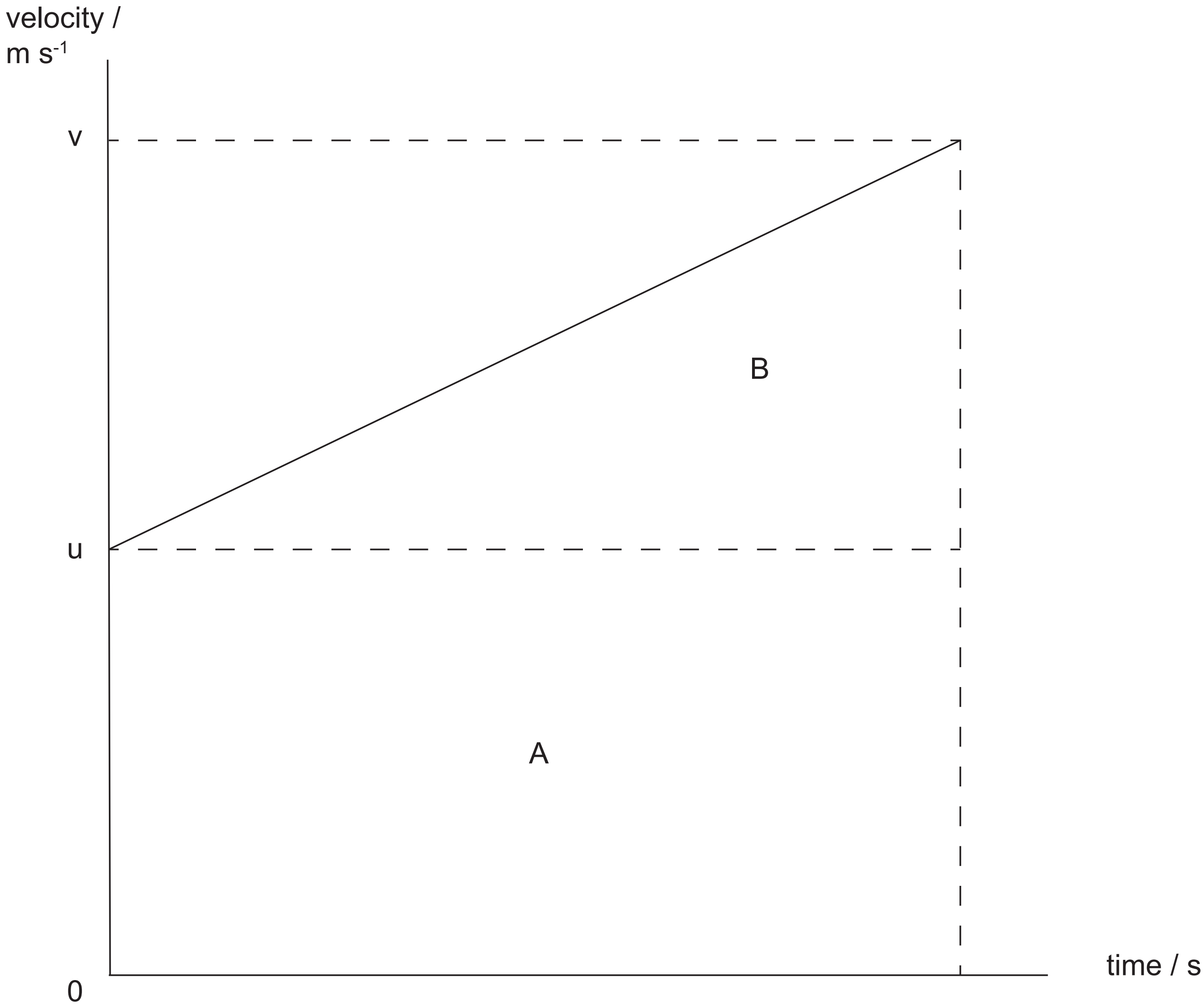
s = ut + $1/2$ (t x at)
s= ut + $1/2$ at^2

$s = \frac{1}{2}(u + v)t$
$s = \frac{1}{2}(u + (u + at))t$
$s = \frac{1}{2}(2u + at)t$
$s = \frac{1}{2}(2ut + at^2)$
$\mathbf{s = ut + \frac{1}{2}at^2}$

(1.4)

An alternative approach to deriving Equation 1.4 is to consider a velocity-time graph.

The following velocity-time graph is for an object accelerating uniformly from *u* to *v* in time *t*.



The displacement s during this acceleration is represented by the area under the velocity-time graph.

s = area under velocity-time graph
s = area A + area B
s = (length x breadth) + $1/2$ (base x height)
s = (t x u) + $1/2$ (t x (v-u))
but we know

$$a = \frac{(v-u)}{t}$$

so *at* = *(v-u)*

This is another important equation for you to be able to apply.

All of the equations we have derived have included time *t*. We will now derive an equation which does not involve *t*. To begin, let us rearrange Equation 1.2 in terms of *t*:

$$\begin{aligned} v &= u + at \\ \therefore v - u &= at \\ \therefore\ t &= \frac{v-u}{a} \end{aligned}$$

We can now substitute for *t* in equation 1.4:

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ \therefore s &= u\frac{(v-u)}{a} + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2 \\ \therefore s &= u\frac{(v-u)}{a} + \frac{1}{2a}(v-u)^2 \\ \therefore 2as &= 2u(v-u) + (v-u)^2 \\ \therefore 2as &= 2uv - 2u^2 + (v^2 - 2uv + u^2) \\ \therefore 2as &= -u^2 + v^2 \\ \therefore v^2 &= u^2 + 2as \end{aligned} \tag{1.5}$$

This is the fourth of the equations you need to be able to apply.

These four equations are called the **kinematic relationships**, and we can use them to solve problems involving motion with constant acceleration. Remember that *s*, *u*, *v* and *a* are all vector quantities.

Always use the same procedure to solve a kinematics problem in one dimension - sketch a diagram, list the data and select the appropriate kinematic relationship.

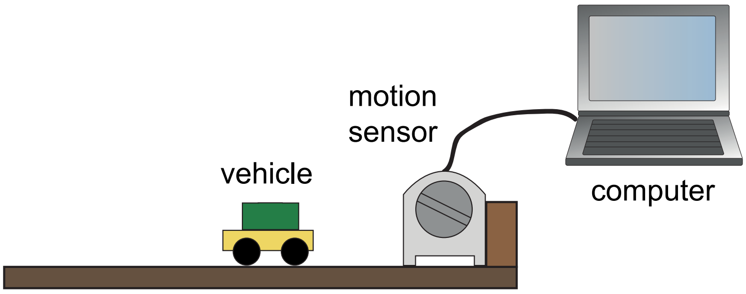
In an assessment you may be asked to apply these relationships.

These relationships are listed on the data sheet that you will be given for all assessments.

It is very unlikely that you will be asked to reproduce one of the derivations given above.

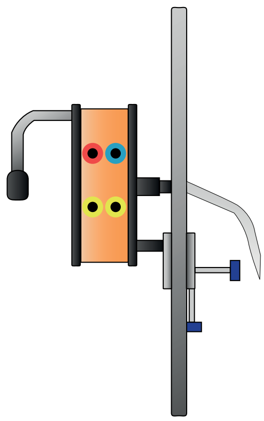
1.4 Measuring displacement, time, velocity and acceleration

The displacement of an object can be measured using a motion sensor.

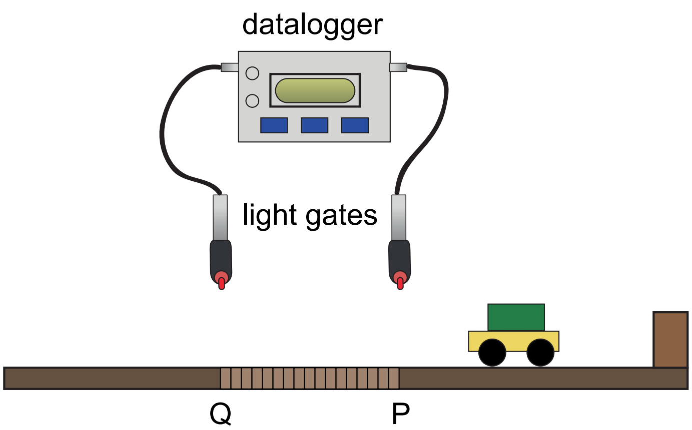


The motion sensor contains an ultrasound transmitter. This sends out pulses of high frequency sound that are reflected back to a special microphone on the sensor. The sensor measures the time for the sound pulses to return to the microphone and uses the speed of sound to calculate the distance of the object from the sensor. This is usually displayed on a computer screen as a displacement time graph.

Light gates are devices that can be used to measure how long it takes an object to pass a point.



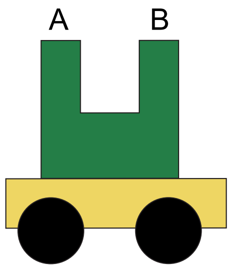
A beam of light passes from the box to a sensor on the arm of the light gate. The light gate is attached to a timer such as a datalogger. When an object breaks the beam the timer measures the amount of time the beam is blocked for. If the length of the object is known then the speed of the object can be calculated, this is often done by the timing device. The following set up can be used to measuring the acceleration of a vehicle on a slope.



Two light gates are placed at P and Q and connected to the datalogger. The length of the car is measured an entered into the datalogger. When the car passes through the light gate at P the datalogger measures the time it takes to pass and then calculates its speed by dividing the length of the car by the time taken, this is the initial velocity, u. It then measures the time taken for the car to go from P to Q, t. Finally it measures the time taken to pass through the light gate at Q and then calculates its speed by dividing the length of the car by the time taken, this is the final velocity, v. The acceleration of the car can be calculated using equation 1.2. a = (v-u)/t. The apparatus shown above can also be used to calculate the acceleration by another method.

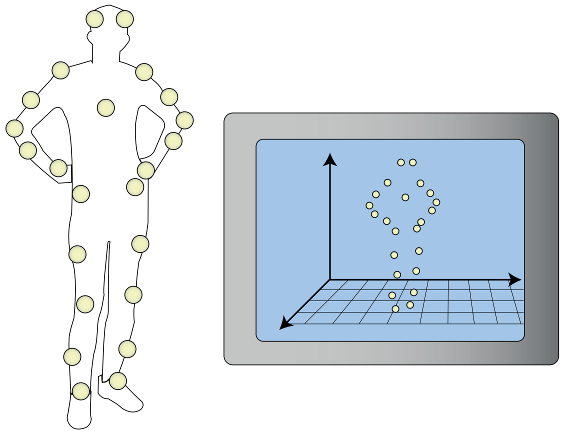
If instead of measuring the time taken to go from light gate P to Q the distance between the two light gates, s, is measured then equation 1.5, ($v^2 = u^2 + 2as$) can be used to calculate the acceleration.

Acceleration can be measured with a single light gate as long as a 'double mask' is used.

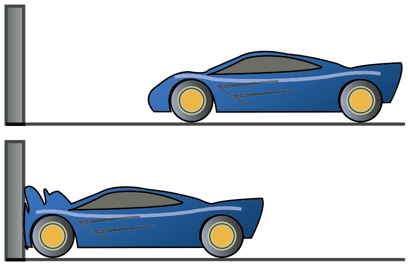


The above mask is used as follows: When the first section of the mask, A, passes through the light gate the datalogger measures the time to pass and then calculates the initial speed, u, by dividing the length of A by the time. The time for the gap to pass through the light gate, t, is recorded. When the final section, B, passes through the light gate the datalogger measures the time to pass and then calculates the final speed, v, by dividing the length of B by the time. The acceleration of the light gate can be calculated using equation 1.2. a = (v-u)/t.

The motion of an object can also be analysed using video techniques. This has become increasingly important in the film and computer areas of the entertainment industry. The illustration below shows how motion capture is used to analyse the movement of a person.



The person has a number of lights placed on their clothes and then stand against a dark background containing a grid. A video is taken of them as they move across the grid and then analysed frame by frame so that their motion can be reproduced in computer graphics. Similar techniques are used when analysing the motion of vehicles in crash test laboratories.

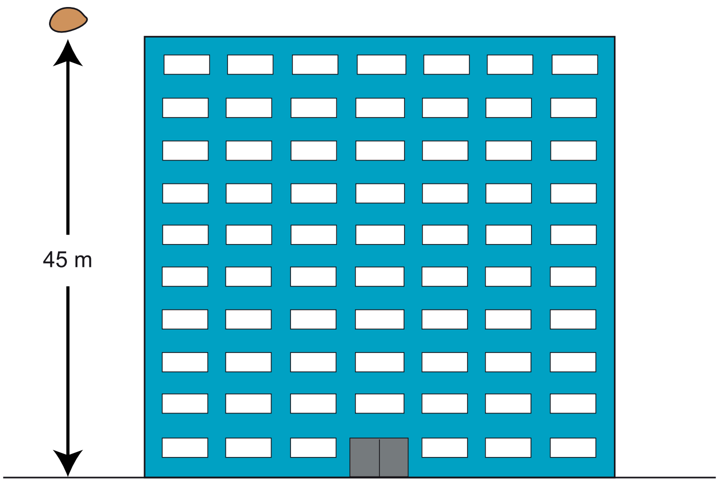


The diagram shows two still frames from the video of a car being crashed at high speed. By making measurements from these frames computer software can be used to analyse the motion of the car. This will allow the acceleration of the car to be calculated and hence the forces acting on occupants of a car in a high speed crash.

Example

A stone is dropped from the top a building which is 45 m high. If the acceleration due to gravity is 9.8 m s^{-2}, calculate

1. the time taken for the stone to reach the ground;
2. the downward velocity of the stone when it hits the ground.



From the question, we are told that the stone is dropped, so its initial velocity u = 0 m s^{-1}.

Its acceleration (downwards) a is 9.8 m s^{-2} and its displacement (downwards) s = 45 m.

1. Given u, a and s, to find t we use the relationship $s = ut + \frac{1}{2}at^2$.

$$s = ut + \frac{1}{2}at^2$$
$$\therefore 45 = 0 + \left(\frac{1}{2} \times 9.8 \times t^2\right)$$
$$\therefore 45 = 4.9t^2$$
$$\therefore t^2 = \frac{45}{4.9} = 9.18$$
$$\therefore t = 3.0 \text{ s}$$

The time taken to fall this distance is 3.0 seconds.

2. To find *v* when we know the values of *u*, *a* and *s*, we use the relationship $v^2 = u^2 + 2as$

$$v^2 = u^2 + 2as$$
$$\therefore v^2 = 0^2 + (2 \times 9.8 \times 45)$$
$$\therefore v^2 = 882$$
$$\therefore = 30 \text{ m s}^{-1} \text{ to the nearest whole number}$$

The final velocity of the object is 30 m s^{-1}

Horizontal Motion Go online

Suppose a car is being driven at a velocity of 12.0 m s^{-1} towards a set of traffic lights, which are changing to red. The car driver applies her brakes when the car is 30.0 m from the stop line. What is the minimum uniform deceleration needed to ensure the car stops at the line?

There is an online activity available which will provide further practice in this type of problem.

Quiz: Kinematic relationships Go online

Useful data:

acceleration due to gravity *g*	9.8 m s^{-2}

Q22: A car accelerates from rest at a rate of 3.5 m s^{-2}. How far has the car travelled after 4.0 s?

a) 7 m
b) 28 m
c) 56 m
d) 78 m
e) 98 m

. . . .

Q23: A ball is thrown vertically upwards. What is the acceleration of the ball when it is at its maximum height?

a) 0 m s^{-2}
b) 9.8 m s^{-1} upwards
c) 9.8 m s^{-1} downwards
d) 9.8 m s^{-2} upwards
e) 9.8 m s^{-2} downwards

. . . .

Q24: In a film stunt, a car is pushed over a cliff, landing on the ground 55 m below the cliff edge. What is the vertical velocity of the car when it strikes the ground?

a) 9.8 m s^{-1}
b) 23 m s^{-1}
c) 33 m s^{-1}
d) 55 m s^{-1}
e) 145 m s^{-1}

. . . .

Q25: When the brakes are applied, a car slows from 25 m s^{-1} to 10 m s^{-1} in 3.6 s. What is the acceleration of the car?

a) -0.10 m s^{-2}
b) -0.24 m s^{-2}
c) -4.2 m s^{-2}
d) -9.7 m s^{-2}
e) -69 m s^{-2}

. . . .

Q26: A cricketer hits a ball at an angle of 75° to the ground. If the ball leaves the bat with velocity 24 m s^{-1}, what is the maximum height above the ground that the ball reaches?

a) 1.2 m
b) 2.0 m
c) 7.6 m
d) 27 m
e) 55 m

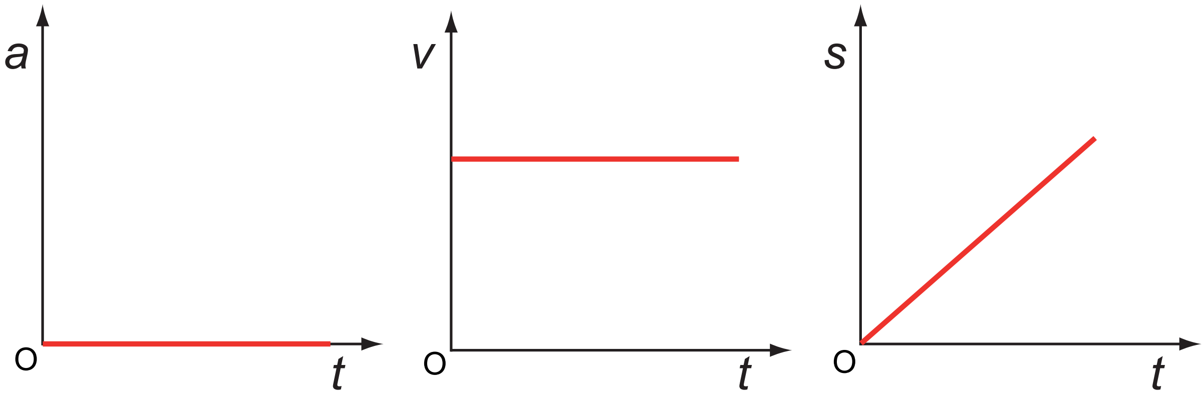
1.5 Acceleration, velocity and displacement time graphs

We will look at how motion with constant acceleration can be represented in graphical form. We can use graphs to show how the acceleration, velocity and displacement of an object vary with time.

Once we have drawn a velocity time graph for an object we can find the acceleration by calculating the gradient of the line on the velocity-time graph. The displacement is found by finding the area under the line on the velocity-time graph. Once these quantities have been found they can be plotted on separate graphs.

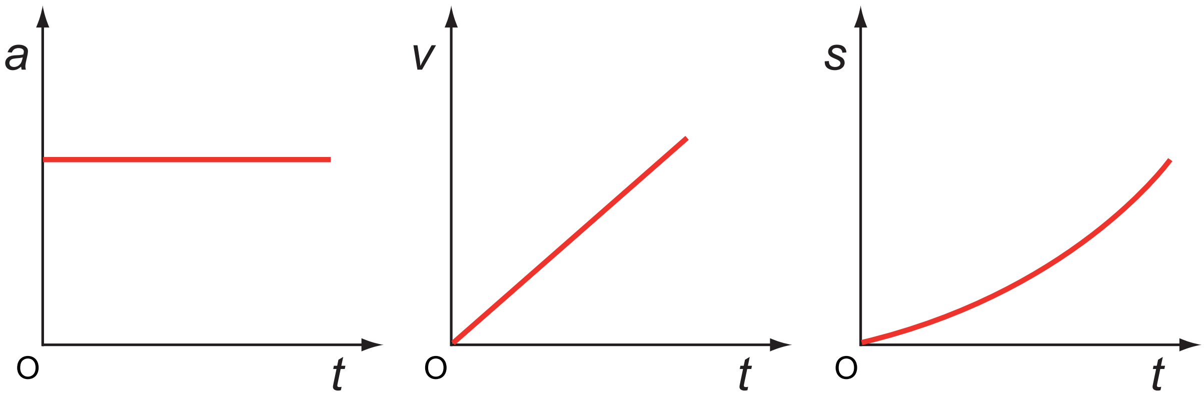
1. Suppose a car is being driven along a straight road at constant velocity. In this case the acceleration of the car is zero at all times, whilst the velocity has a constant value.

Figure 1.10: Graphs for motion with constant velocity



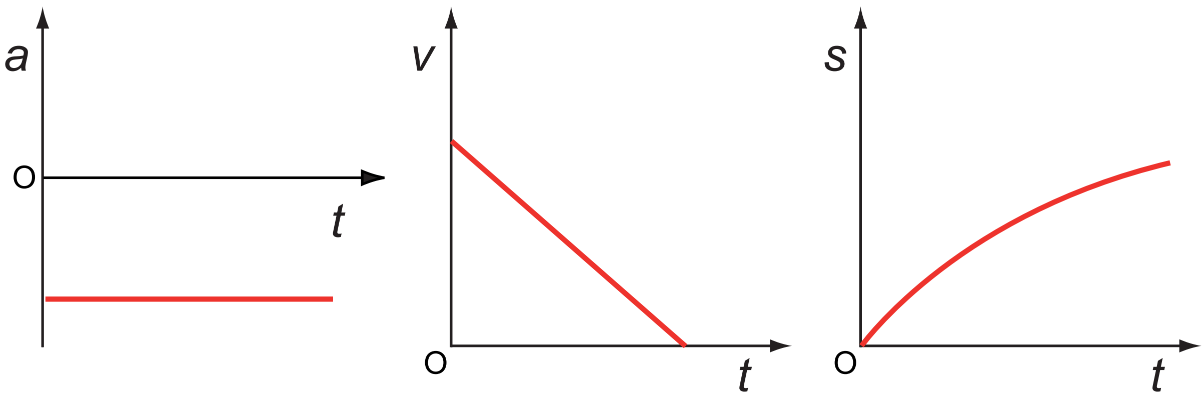
2. Suppose instead the car starts from rest, accelerating at a uniform rate of, say, 2.0 m s^{-2}. The velocity increases by 2.0 m s^{-1} every second.

Figure 1.11: Graphs for motion with constant positive acceleration



3. What if the car is travelling at a certain velocity when the brakes are applied? In this case the car may be decelerating at 2.0 m s^{-2} (an acceleration of -2.0 m s^{-2}) until it comes to rest.

Figure 1.12: Graphs for motion with constant negative acceleration



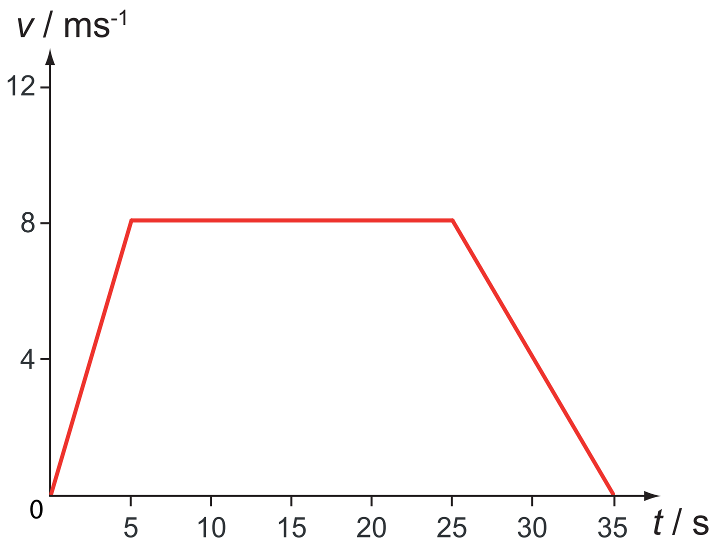
Acceleration Go online

There is an online activity at this stage exploring these relationships.

Example

The graph in Figure 1.13 shows the motion of a car. The car starts from rest, accelerating uniformly for the first 5 s. It then travels at constant velocity of 8.0 m s^{-1} for 20 s, before the brakes are applied and the car comes to rest uniformly in a further 10 s.

Figure 1.13: Velocity-time graph



1. Using the graph in Figure 1.13, calculate the value of the acceleration of the car whilst the brakes are being applied.
2. Sketch the acceleration-time graph for the motion of the car.

1. The graph shows us that the car slows from 8.0 m s^{-1} to rest in 10 s. Its acceleration over this period is:

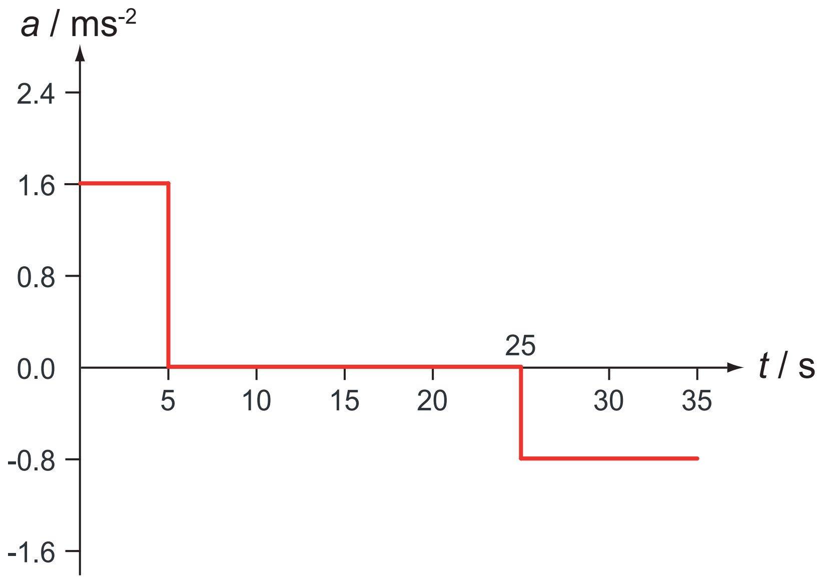
$$\begin{aligned} a &= \frac{v - u}{t} \\ &= \frac{0 - 8}{10} \\ &= -0.8 \text{ m s}^{-2} \end{aligned}$$

2. The car starts from rest, and accelerates to a velocity of 8.0 m s^{-1} in 5.0 s. The acceleration *a* over this period is:

$$
\begin{aligned}
a &= \frac{v - u}{t} \\
&= \frac{8 - 0}{5} \\
&= 1.6\,\text{m s}^{-2}
\end{aligned}
$$

For the next 20 s, the car is travelling at constant velocity, so its acceleration is zero. We have already calculated the acceleration between *t* = 25 s and *t* = 35 s. Using the values of acceleration we have just calculated, the acceleration-time graph can be plotted:

Figure 1.14: Acceleration-time graph

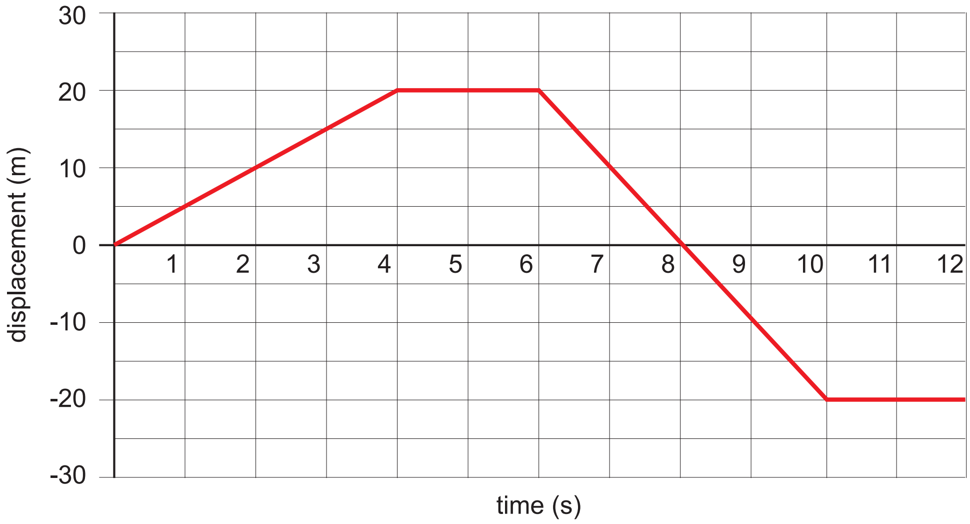


1.5.1 Interpreting graphs

Extra Help: Interpretation of graphs

Activity 1: Motion with uniform velocities

Displacement - time graph (uniform velocities)



Now attempt the following questions which are based on the above graph.

Q27: What change in displacement is there between 0 s and 2 s? (m)

. . . .

Q28: What change in displacement is there between 2 s and 4 s? (m)

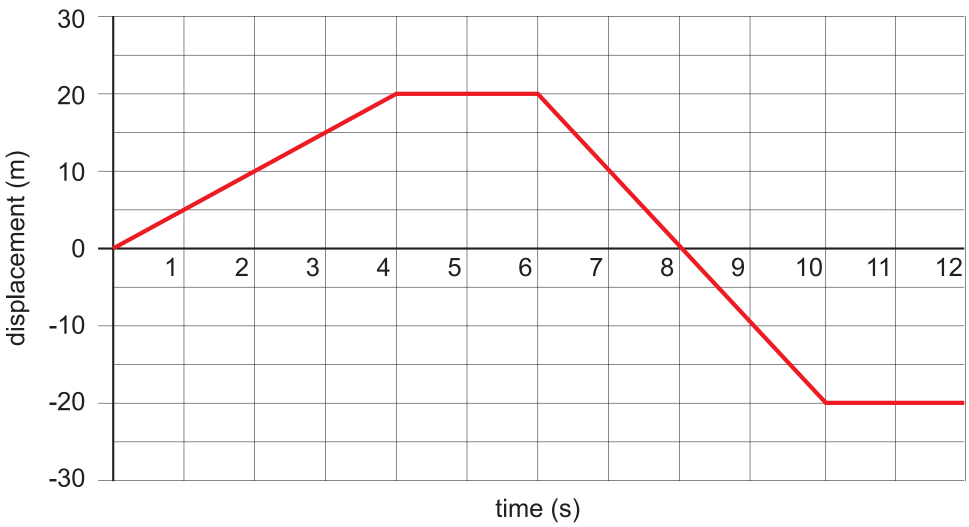
. . . .

Q29: What do these two answers tell you about the velocity during the each of these 2 s intervals? The velocity is increasing/decreasing/constant, (select correct word).

. . . .

Q30: Calculate the value of the velocity between 0 s and 4 s. (m s^{-1})

Displacement - time graph (uniform velocities)



Q31: What change in displacement is there between 4 s and 5 s? (m)

. . . .

Q32: What change in displacement is there between 5 s and 6 s? (m)

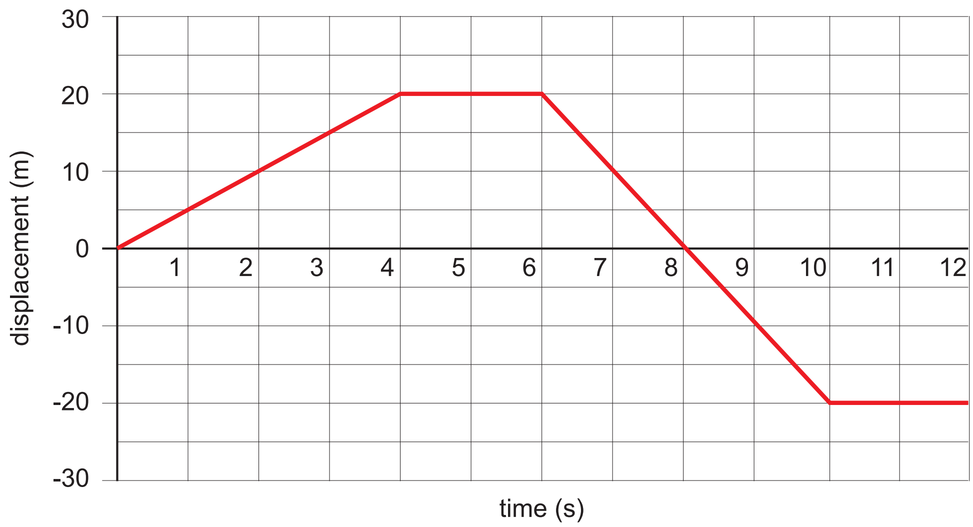
. . . .

Q33: What do these two answers tell you about the velocity during the each of these one second intervals? The velocity is increasing/decreasing/constant, (select correct word).

. . . .

Q34: Calculate the value of the velocity between 4 s and 6 s. (m s^{-1})

Displacement - time graph (uniform velocities)



Q35: What change in displacement is there between 6 s and 8 s? (m)

. . . .

Q36: What change in displacement is there between 8 s and 10 s? (m)

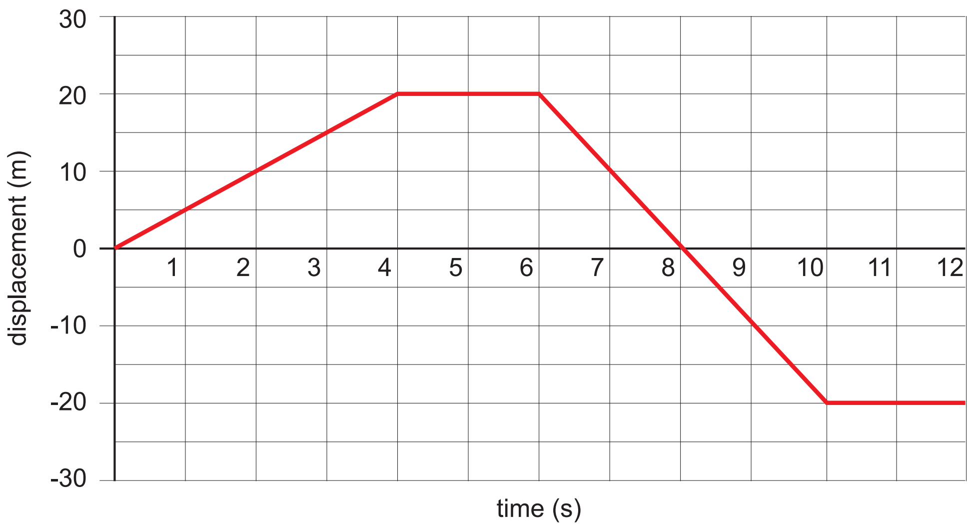
. . . .

Q37: What do these two answers tell you about the velocity during the each of these 2s intervals? The velocity is increasing/decreasing/constant, (select correct word).

. . . .

Q38: Calculate the value of the velocity between 6 s and 10 s. (m s^{-1})

Displacement - time graph (uniform velocities)



Q39: What change in displacement is there between 10 s and 11 s? (m)

. . . .

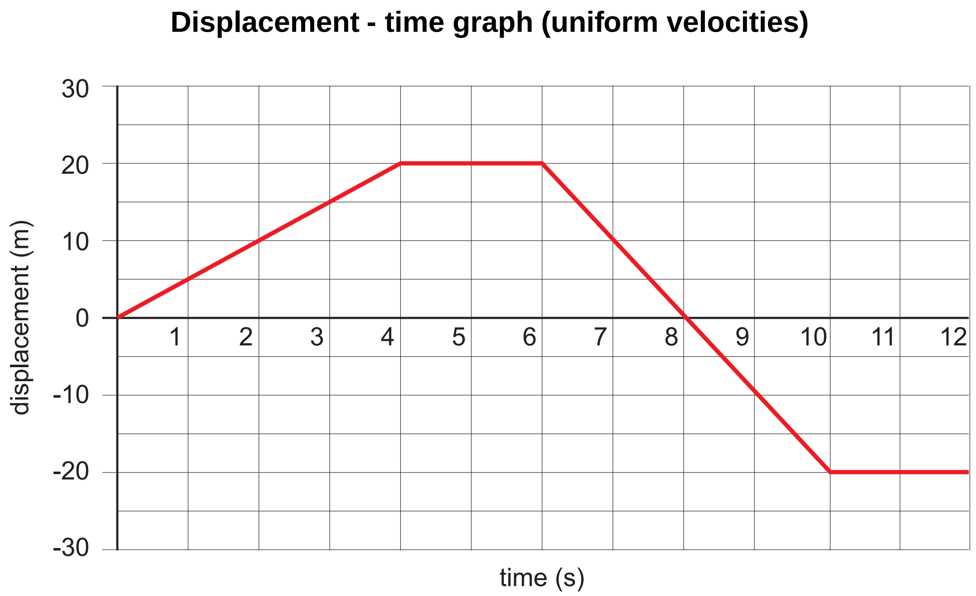
Q40: What change in displacement is there between 11 s and 12 s? (m)

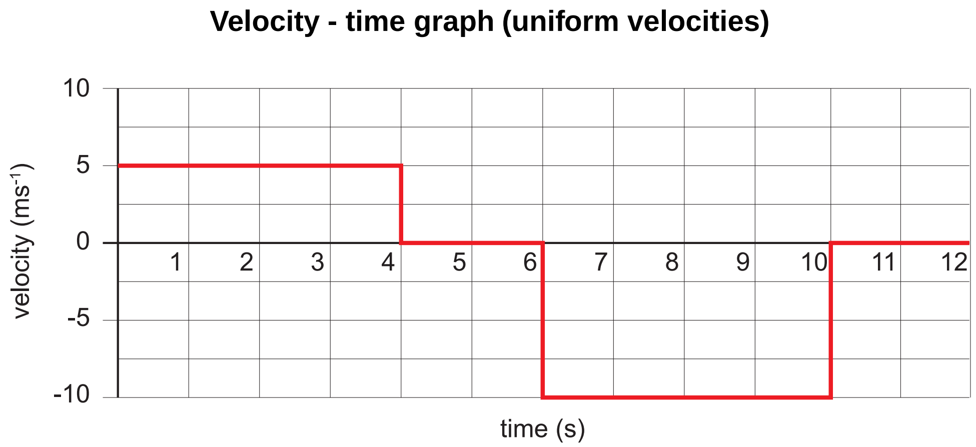
. . . .

Q41: What do these two answers tell you about the velocity during the each of these one second intervals? The velocity is increasing/decreasing/constant, (select correct word).

. . . .

Q42: Calculate the value of the velocity between 10 s and 12 s. ($m\ s^{-1}$)



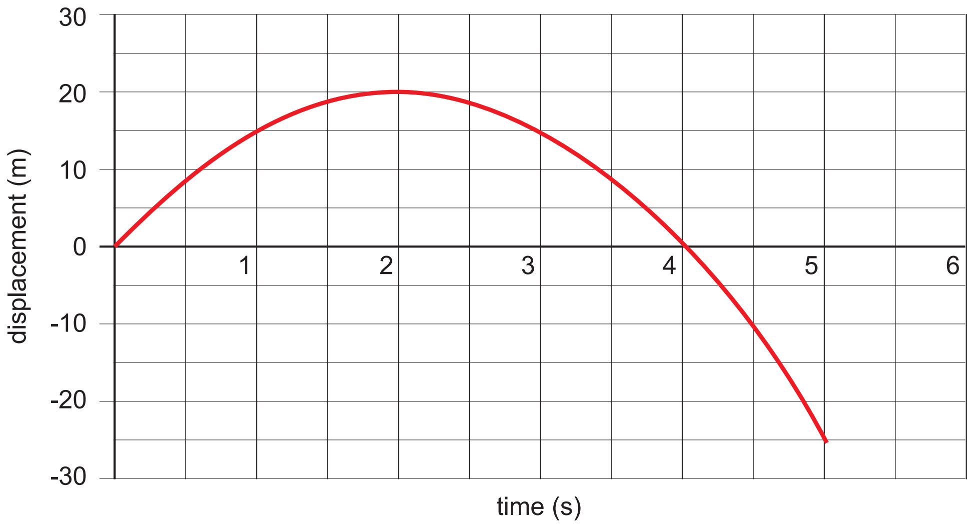


Notice the following:

- when the displacement time graph has a positive gradient (line slopes up) the velocity is positive
- when the displacement time graph has a gradient of zero (horizontal line) the velocity is zero
- when the displacement time graph has a negative gradient (line slopes down) the velocity is negative.

Activity 2: Motion with uniform acceleration

Displacement - time graph (uniform acceleration)



Now attempt the following questions which are based on the above graph.

Q43: What change in displacement is there between 0 s and 1 s? (m)

. . . .

Q44: What change in displacement is there between 1 s and 2 s? (m)

. . . .

Q45: What do these two answers tell you about the velocity during the each of these one second intervals? The ball is moving with an acceleration/constant velocity. (select correct word).

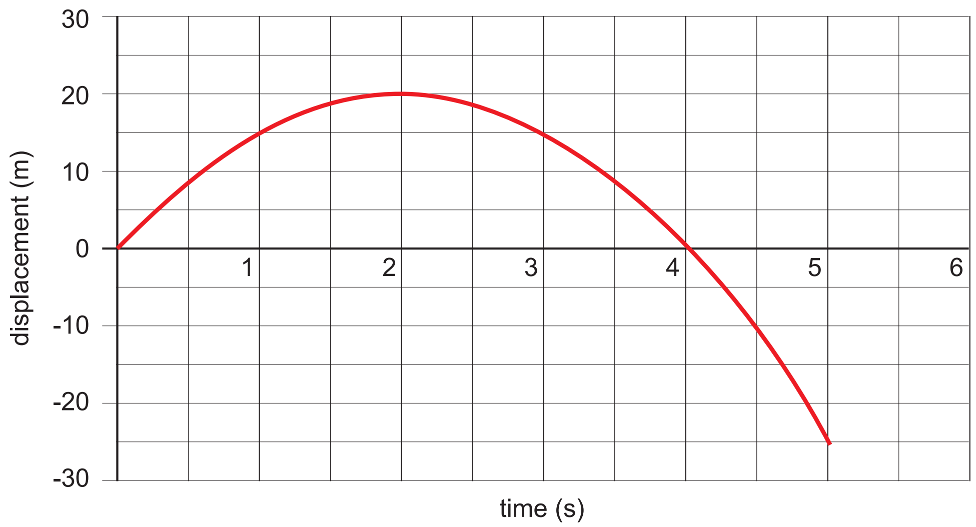
. . . .

Q46: iv. The average velocity between 0 s and 1 s is 15 m s^{-1} and between 1 s and 2 s is 5 m s^{-1}. Using the equation

$$a = \frac{(v - u)}{t}$$

calculate the acceleration of the ball between these one second intervals. (m s^{-2})

Displacement - time graph (uniform acceleration)



Q47: What change in displacement is there between 2 s and 3 s? (m)

. . . .

Q48: What change in displacement is there between 3 s and 4 s? (m)

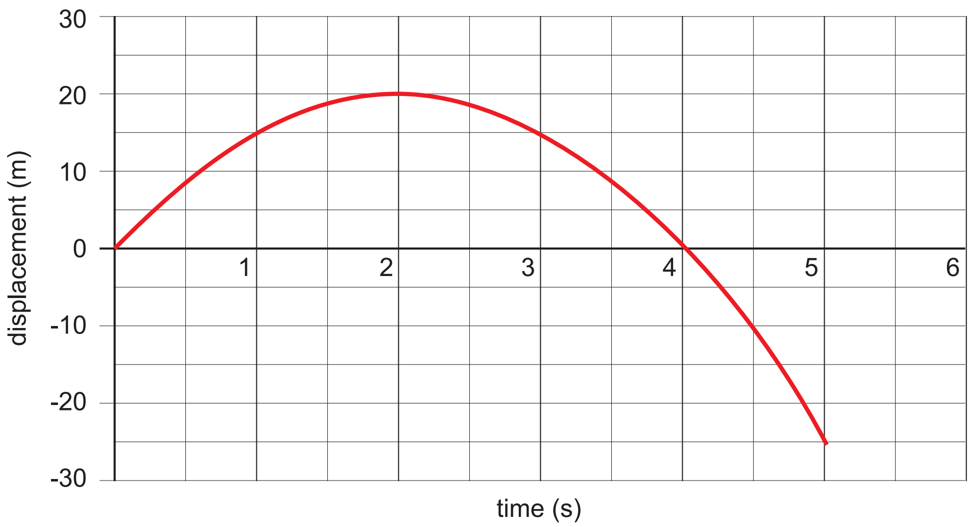
. . . .

Q49: What do these two answers tell you about the velocity during each of these one second intervals? The ball is moving with an acceleration/constant velocity. (select correct word).

. . . .

Q50: Calculate the acceleration between these two time intervals. (m s^{-2})

Displacement - time graph (uniform acceleration)



Q51: What change in displacement is there between 3 s and 4 s? (m)

. . . .

Q52: What change in displacement is there between 4 s and 5 s? (m)

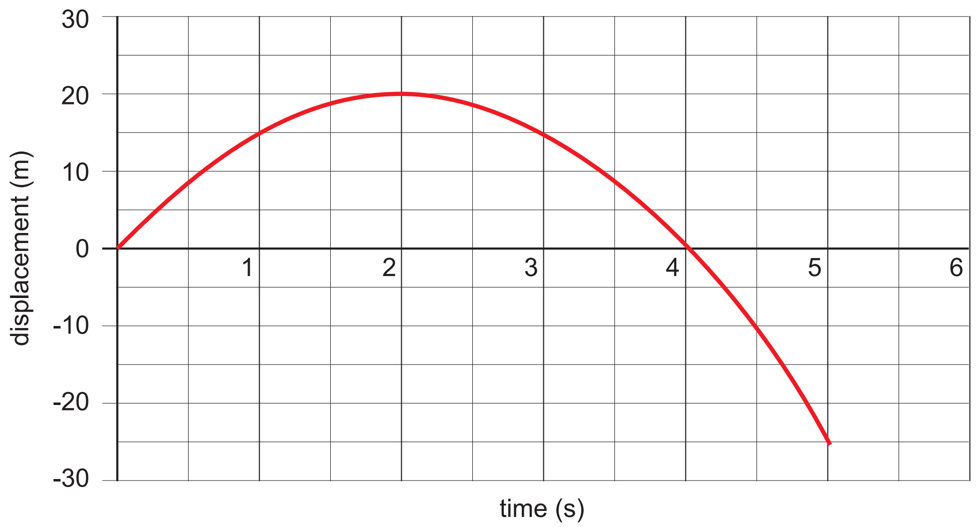
. . . .

Q53: What do these two answers tell you about the velocity during each of these one second intervals? The ball is moving with an acceleration/constant velocity. (select correct word).

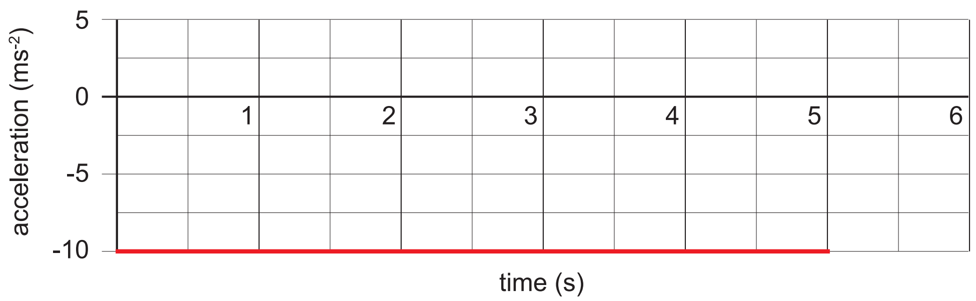
. . . .

Q54: Calculate the acceleration between these two time intervals. (m s^{-2})

Displacement - time graph (uniform acceleration)



Displacement - time graph (uniform acceleration)



Notice the following:

- when the displacement time graph has a positive value, the ball is above the starting point
- when the displacement time graph has a negative value the ball is the below starting point
- in this example the acceleration is constant, see answers to the 4th question in each set of questions
- the acceleration of the ball is due to gravity so is in a downward direction and must therefore have a negative value.

1.5.2 Motion of a bouncing ball

Motion of a bouncing ball Go online

What do the acceleration-time and velocity-time graphs look like for the motion of a bouncing ball?

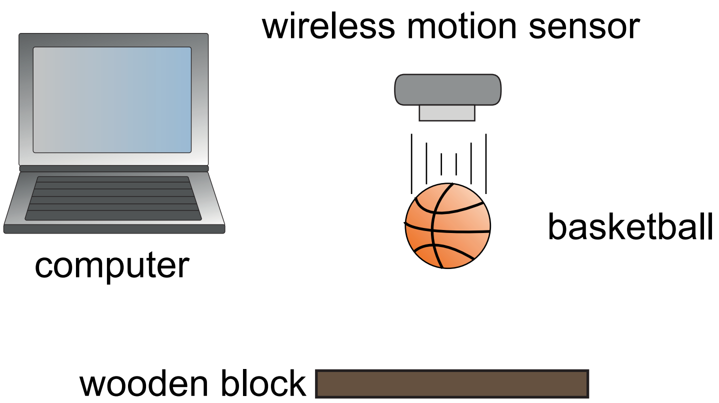
The motion of a bouncing ball can be described using the kinematic relationships.

Try to answer the following questions:

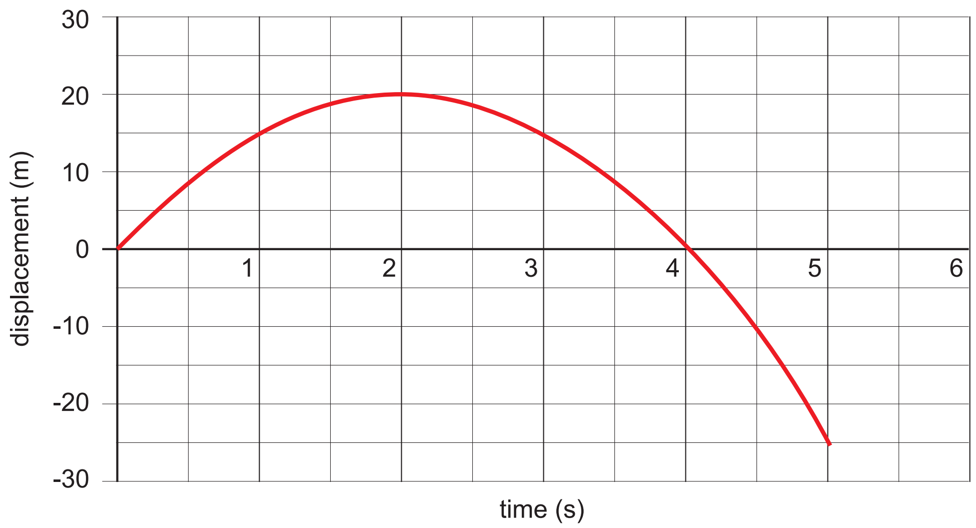
1. Suppose the ball is dropped from a height of 10 m. What is its velocity just before it hits the ground? (Remember the sign convention.)
2. A ball dropped from a height of 10 m rebounds with initial velocity 12 m s^{-1}. To what height does it rise?
3. In which direction is the ball moving when the velocity is negative?
4. In which direction is the ball moving when the velocity is positive?
5. What is happening to the direction of movement of the ball when its velocity changes sign?

At this stage there is an online activity. If however you do not have access to the internet you read the following and ensure that you understand it.

One way of examining the motion of a bouncing ball is to use a motion sensor. The apparatus shown below is set up.



The ball is allowed to drop and the computer displays the following graph.



We can see from the displacement time graph that the ball has been dropped from 2 m below the motion sensor. The ball hits the block at 0.6 s and is in contact with it for 0.1 s. After that it rebounds and starts to rise again making the displacement decrease.

If the ball was dropped on to a softer surface the resulting graph would be different. The time of contact would be greater than 0.1 s and it would not rise to such a great height on rebound.

1.5.3 Motion on a slope

Motion of toy car moving down a slope and hitting a stretched elastic band Go online

This interactivity is only online.

1.5.4 Quiz: Acceleration

Quiz: Acceleration Go online

Q55: An Olympic sprinter accelerates from rest to a velocity of 9.0 m s^{-1} in the first 2.5 s of a race. What is the average acceleration of the sprinter during this time?

a) 0.28 m s^{-2}
b) 0.69 m s^{-2}
c) 1.44 m s^{-2}
d) 3.6 m s^{-2}
e) 22.5 m s^{-2}

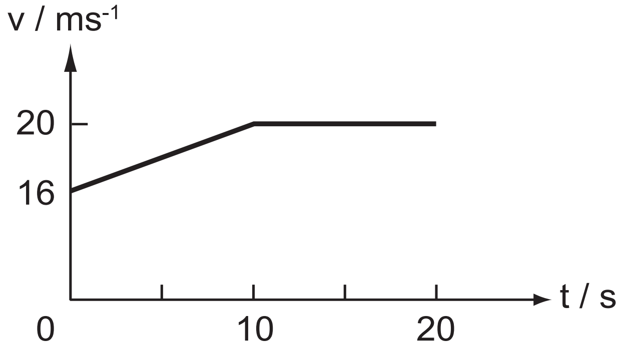
. . . .

Q56: A motorcycle is travelling at a constant velocity along a straight road. Which *one* of the following statements is true?

a) Both the acceleration and the velocity of the motorcycle have constant, positive values.
b) The velocity of the motorcycle is constant, so its acceleration is zero.
c) The velocity of the motorcycle is positive, but its acceleration is negative.
d) The acceleration of the motorcycle increases with time.
e) The acceleration of the motorcycle decreases with time.

. . . .

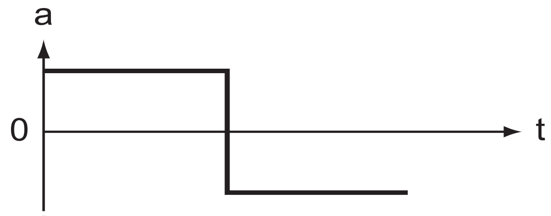
Q57: A car is being driven along a straight road. The velocity-time graph is shown below. What is the acceleration of the car between t = 0 and t = 10 s?

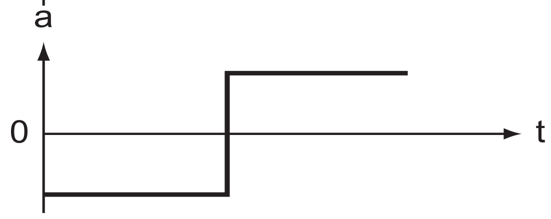


a) 0.4 m s^{-2}
b) 0.5 m s^{-2}
c) 1.6 m s^{-2}
d) 2.0 m s^{-2}
e) 2.5 m s^{-2}

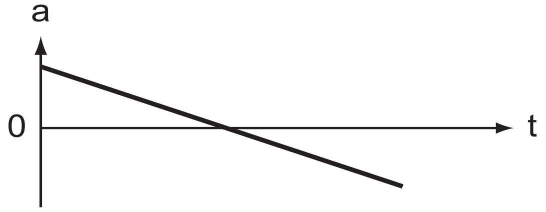
. . . .

Q58: A tennis ball is thrown vertically upwards. It reaches a height of 5.0 m before falling back to the ground. Which of the following could represent the acceleration-time graph of the ball during the whole of this motion?

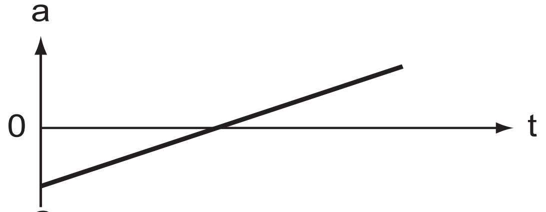
a)


b)


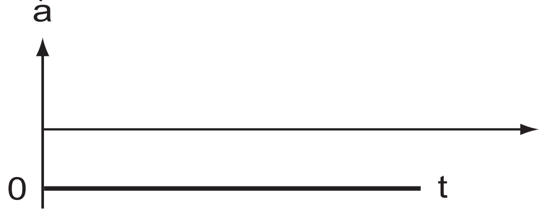
c)



d)

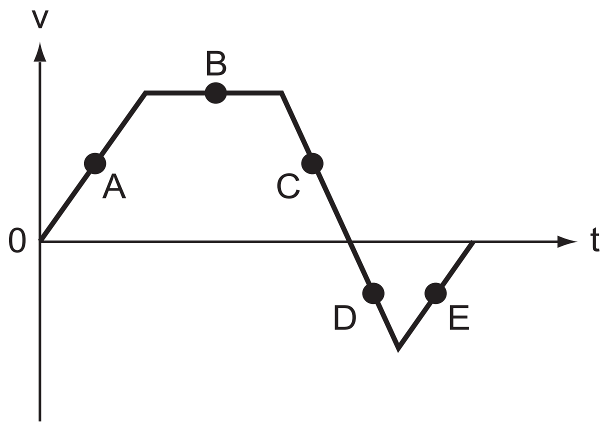


e)



....

Q59: The following graph shows the velocity against time for a certain object. At which points is the object moving with positive acceleration?



a) A and B
b) A and E
c) C and D
d) A and C
e) B and E

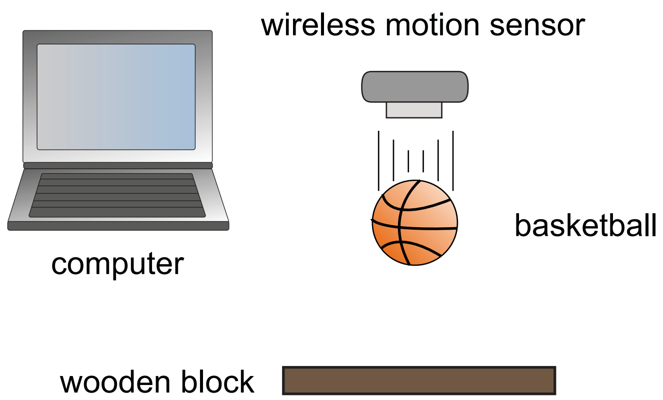
1.6 Motion of objects

Up till now you have been examining how objects move. You used a series of equations to investigate the motion of objects with a constant acceleration. You used graphs to analyse the movement of objects.

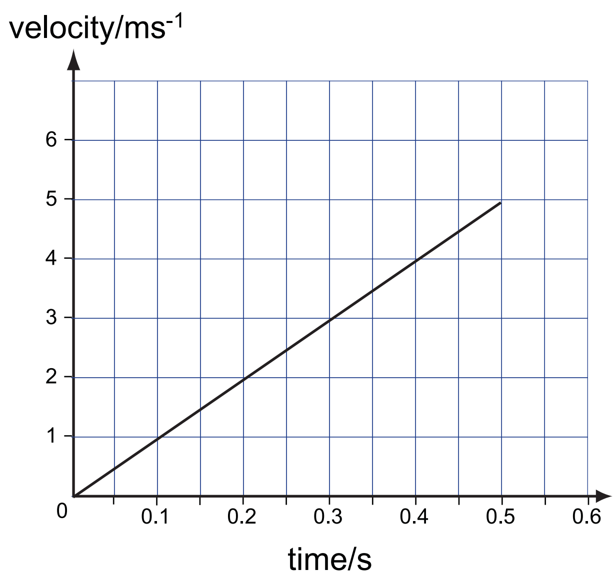
In this topic we will look at two particular cases of accelerating objects: an object in freefall and an object accelerating down a slope.

1.6.1 Freefall

If an object is held above the Earth and released it falls. This is because it is accelerating towards the Earth. In the absence of air friction all objects have the same acceleration as they fall towards the surface of the Earth. If a ball is dropped near the surface of the Earth a motion sensor can be used to measure its speed at different times.



The data can be used to produce a speed time graph and the acceleration calculated from the gradient of the line.



When we determine the gradient of this line we find that it has a value of 9.8 m s^{-2}. In the absence of air friction we get the same result for any object dropped near the surface of the Earth. We can use this to define the weight of an object. The weight of an object on Earth is the force it exerts due to the gravitational pull of the Earth. The weight is a force and so is measured in Newtons. It can be calculated by multiplying the mass of the object by the gravitational acceleration of the Earth.

W = mg

Where W is the weight in Newton's, m is the mass in kilograms and g is the acceleration due to gravity in metres per second squared.

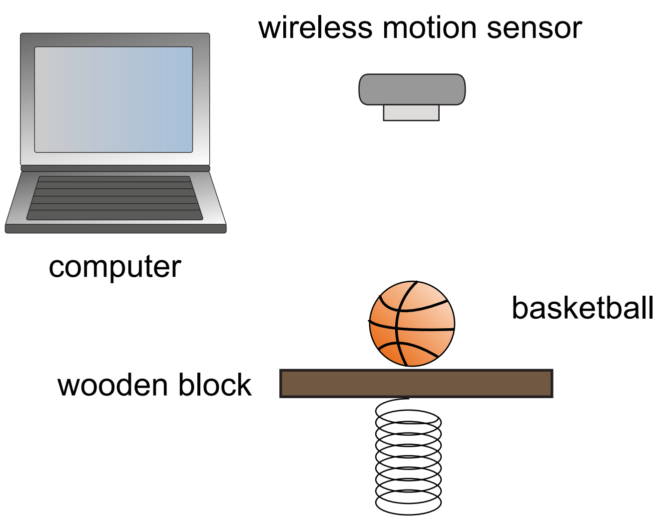
Freefall Go online

Q60: Using the sample data, produce a graph and calculate the value of *g*.

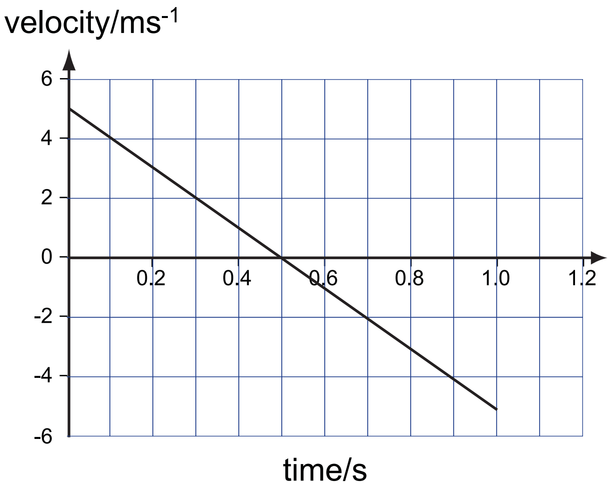
time/s	velocity/m s^{-1}
0	0
0.1	0.98
0.2	1.96
0.3	2.94
0.4	3.92
0.5	4.9

1.7 An object projected vertically upwards

A ball is projected upwards from the ground using a spring towards a motion sensor.



The motion sensor and computer software produce the following results.



In graphs of this type the initial direction of movement is often taken as positive. This is very important when the moving object will change direction during its motion.

In this graph the ball is projected upwards so upward velocities are positive and downwards velocities are negative.

We can draw a number of conclusions from this graph.

The velocity initially is positive but it becomes a negative value after 0.5 s. This means that the ball has changed direction of movement at 0.5 s.

The graph has only one single straight line. This means that the gradient of the line is constant. As the gradient of the line is the acceleration of the object we can see that the object has a constant acceleration throughout its motion. The gradient of this graph is - 9.8 m s^{-2}. The negative sign indicates that the direction of acceleration is downwards.

The acceleration due to gravity, on Earth, is always 9.8 m s^{-2} vertically downwards as long as there are no other forces acting on the object.

When the object is at its maximum height its velocity is 0 m s^{-1}. It is important to realise that the object has not become stationary, only that its instantaneous velocity is 0 m s^{-1}. The total displacement (the area under the graph) is zero once the object has completed its motion. That is because it has finished off where it started.

Example : The tennis player's serve

The ball toss of a tennis player's serve is very important.

A tennis player throws a ball vertically into the air. The ball will rise and then fall. The player will hit the ball when it is back at its release point.

The player hits the ball 0.6 s after it was thrown vertically into the air. Calculate how high the ball should be thrown. Ignore any frictional forces.

If the total time of flight of the ball is 0.6 s then it must go up for 0.3 s before falling back down for 0.3 s. As it starts to fall its initial velocity is 0 m s^{-1} and its acceleration is the acceleration due to gravity, 9.8 m s^{-2}.

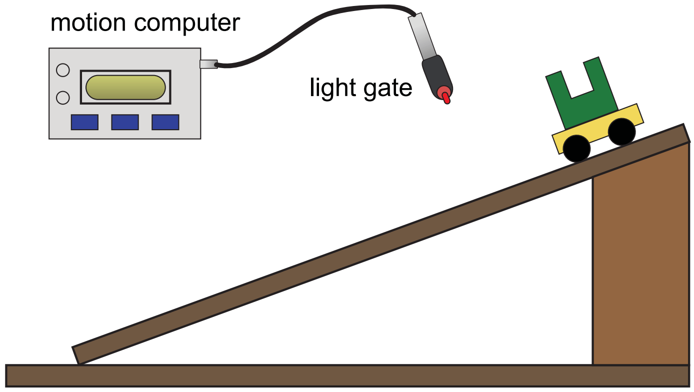
We can use the equation $s = ut + {}^{1}/_{2}at^{2}$ to find the height (displacement) of the ball.

$s = ut + {}^{1}/_{2}at^{2}$
$s = (0 \times 0.3) + {}^{1}/_{2} \times 9.8 \times 0.3^{2} s = 0.44 m$

Therefore if a tennis player is using a timing of 0.6 s between the toss of the ball and striking it he should throw it 0.44 m high.

1.8 An object accelerating down a slope

If an object is released on a frictionless slope it will start to move down the slope. This is because it is being accelerated by the pull of the Earth.



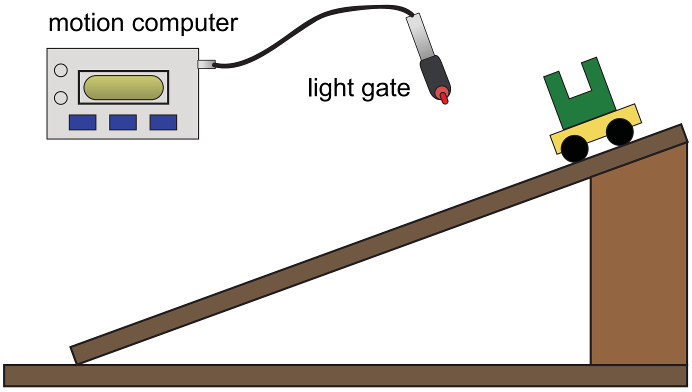
The acceleration of the object will be less than 9.8 m s^{-2}.

If we increase the angle between the slope and the horizontal the acceleration will increase. The online activity that follows allows you to investigate the relationship between the angle of the slope and the acceleration.

Acceleration on a slope Go online

There is an online activity at this stage. If however, you do not have access to the web you should try the following question.

Q61: An experiment is set up to measure the effect on acceleration of an object sliding down a slope.



The system is frictionless.

The formula for calculating the acceleration is a = 9.8 $\times$ sine of the angle of slope.

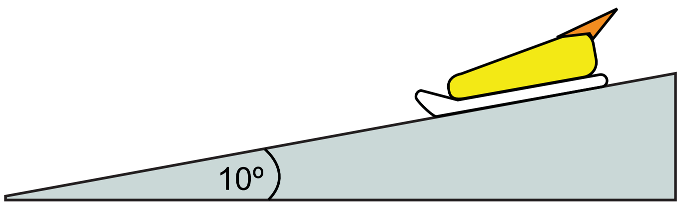
If the slope is adjust in steps of 2° from 0° to 40°, calculate the acceleration in each case entering the results in a table.

Plot a graph of the results.

1. What is the relationship between angle and acceleration?
2. Is acceleration directly proportional to angle?

Example : The bobsleigh track

A bobsleigh track, which can be assumed to be frictionless, contains a section that is at an angle of 10° to the horizontal.



We can calculate the acceleration of the bobsleigh using what we discovered in the activity above. The acceleration is a = 9.8 sin (angle of slope)

Therefore the acceleration is

a = 9.8 sin 10
so a = 1.7 m s^{-2}.

The acceleration of the bobsleigh is 1.7 m s^{-2}.

When designing tracks engineers think very carefully about the angle of slopes on these tracks. If the slopes are too steep then the acceleration of the bobsleighs will be too large and they will become difficult to control. If they are not large enough the races will not be challenging enough.

1.9 Summary

Summary

You should now be able to:

- define acceleration as the rate of change of velocity;
- identify the four kinematic relationships that can be applied to motions with constant acceleration:

$$v = u + at$$
$$s = \tfrac{1}{2}(u + v)t$$
$$s = ut + \tfrac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

- carry out calculations involving motion in one and two dimensions using these kinematic relationships;
- distinguish between distance and displacement, and between speed and velocity;
- define and classify vector and scalar quantities;
- explain what is meant by the resultant of a number of vectors;
- use scale diagrams to find the magnitude and direction of the resultant of a number of vectors;
- carry out calculations to find the perpendicular components of a vector;
- draw acceleration-time, displacement-time graphs and velocity-time graphs, and use them to deduce information about the motion of an object;
- calculate the acceleration of an object falling near the Earth's surface from data on a velocity-time graph;
- investigate how the angle of a frictionless slope affects the acceleration of an object.

1.10 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

Links

- Physics Lab provides a different approach to proving the equations of motion:
http://dev.physicslab.org/Document.aspx?doctype=3&filename=Kinematics_DerivationKinematicsEquations.xml
- This site provides a range of good materials to back up the work of this topic. This page gives a basic definition of acceleration:
http://www.physicsclassroom.com/class/1dkin/u1l1e.cfm
- This is the same site but gives a good explanation of scalars and vectors:
http://www.physicsclassroom.com/class/1dkin/U1L1b.cfm
- The site is clearly laid out for vectors although the symbolism may be different:
http://www.physics.uoguelph.ca/tutorials/vectors/vectors.html
- This site clearly demonstrates the addition of vectors nose-to-tail:
http://www.walter-fendt.de/html5/phen/resultant_en.htm
- There is a good step by step guide on this page about acceleration for solving problems:
http://www.sparknotes.com/testprep/books/sat2/physics/chapter5section4.rhtml

1.11 Assessment

End of topic 1 test Go online

The following test contains questions covering the work from this topic.

The following data should be used when required:
Acceleration due to gravity g = 9.8 m s^{-2}

The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Q62: A car is travelling with velocity m s $^{-1}$. 8.0 s later, the velocity of the car is 10m s^{-1}.

Calculate the average acceleration of the car over this period, in m s $^{-2}$.

. . . .

Q63: A car is travelling at 12 m s *-1* when the brakes are applied. If the car decelerates at a rate of 5.5 m s $^{-2}$, calculate how far the car travels from the time the brakes are applied until the car comes to rest.

. . . .

Q64: A workman fixing a TV aerial to the roof of a building drops one of his tools, which lands on the ground 2.5 s later.

Calculate the height of the workman above the ground, in m.

. . . .

Q65: A car is being driven at a steady velocity of 11 m s $^{-1}$. The driver then accelerates at 2.0 m s *-2* for 3.2s. Calculate the final velocity of the car, in m s $^{-1}$.

. . . .

Q66: A small pebble is knocked off the roof of a building which is 37 m tall.

1. Calculate how much time (in s) elapses before the pebble hits the ground.
2. Calculate the velocity in m s $^{-1}$ of the pebble as it passes a third floor window m above the ground.

. . . .

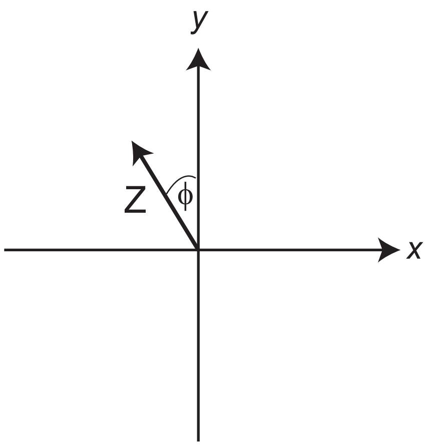
Q67: Two forces A and B act at right angles. Force A has magnitude 45 N and force B has magnitude 30 N. Calculate the magnitude of the resultant force.

. . . .

Q68: A force of 76 N acts in the positive x-y direction, at an angle of 56° to the x-axis. Calculate the magnitude of the component of the force acting in the x-direction.

. . . .

Q69: Consider the vector z shown in the diagram below.

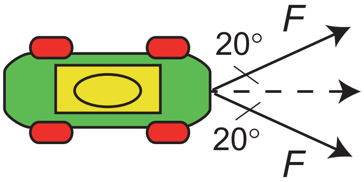


z has magnitude 56 N, and acts at an angle θ = 27° to the y-axis.

1. Calculate the magnitude of the component of z acting in the y-direction.
2. Calculate the magnitude of the component of z acting in the x-direction.

. . . .

Q70: Two ropes are attached to a car which is stuck in a muddy field. The same force F is applied to both ropes, in order to pull the car in the direction shown by the dashed line in the figure below.



Both ropes make an angle of 20° with the dashed line.

If the force F applied along each rope is 320 N, calculate the resultant force in the direction shown by the dashed line.

. . . .

Q71: Which of the following pairs of physical quantities are **both** vector quantities?

a) Distance and velocity
b) Force and temperature
c) Displacement and force
d) Speed and acceleration

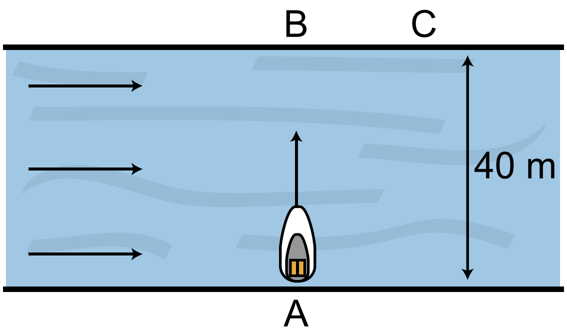
. . . .

Q72: Two forces M and N act in a direction due north. M has magnitude 20 N, and N has magnitude 44 N.

Calculate the magnitude of the force which must act due south to produce a resultant force of magnitude zero.

. . . .

Q73: A speed boat is crossing a river. The boat starts from point A, and the driver points the boat at point B, directly across the river from point A. The river is 40 m wide.

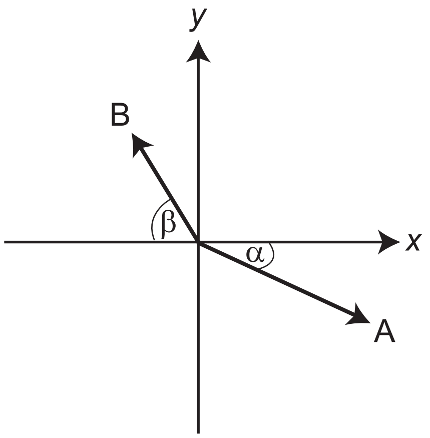


The boat has velocity 7.8 m s^{-1} in the direction AB. The river is flowing at velocity 3.4 m s^{-1}, so that the boat actually arrives at point C, downstream of point B.

1. Calculate the magnitude of the velocity of the boat in the direction AC.
2. Calculate the distance BC.

. . . .

Q74: The two forces A and B shown in the diagram below act on an object placed at the origin.

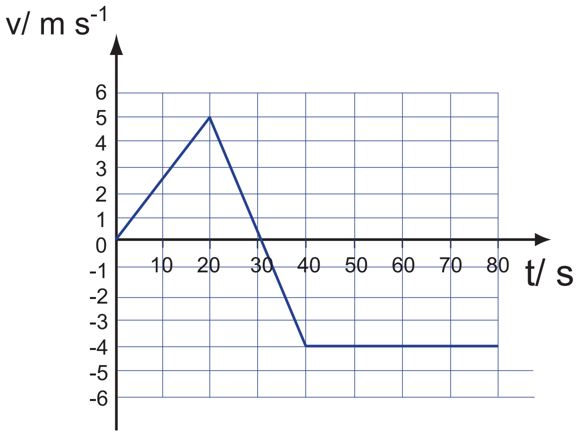


A has magnitude 16 N and acts at angle α = 32° to the x-axis. B has magnitude 3.7 N and acts at angle β = 75° to the x-axis.

1. Calculate the component of the resultant force which acts in the y-direction.
2. Calculate the component of the resultant force which acts in the x-direction.

. . . .

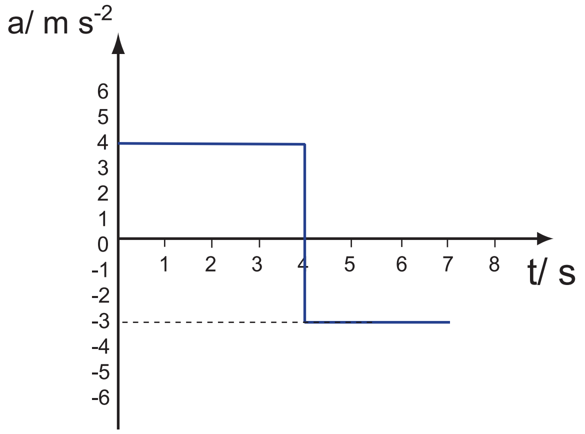
Q75: Consider the following velocity-time graph for the motion of an object.



1. State the value of the acceleration in m s $^{-2}$ of the object at time t = 10 s.
2. State the displacement in m of the object at t = 20 s.
3. State the acceleration in m s $^{-2}$ of the object at t = 30 s.
4. State the acceleration in m s $^{-2}$ of the object at t = 60 s.

. . . .

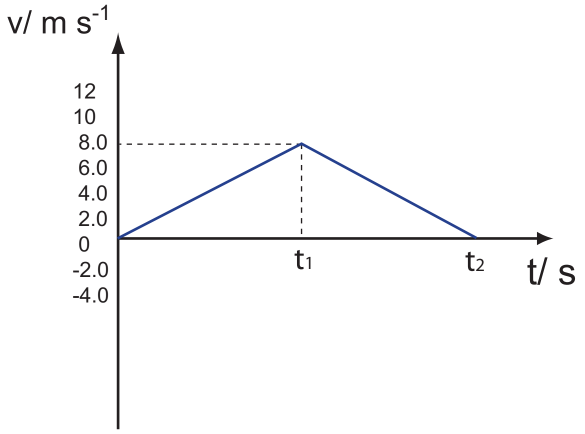
Q76: The acceleration-time graph for an object starting from rest is shown below:



1. Calculate the velocity (in m s $^{-1}$) of the object when t = s.
2. The velocity of the object at t = 5.0 s is 13 m s^{-1} . What is the velocity (in m s $^{-1}$) at t = s?

. . . .

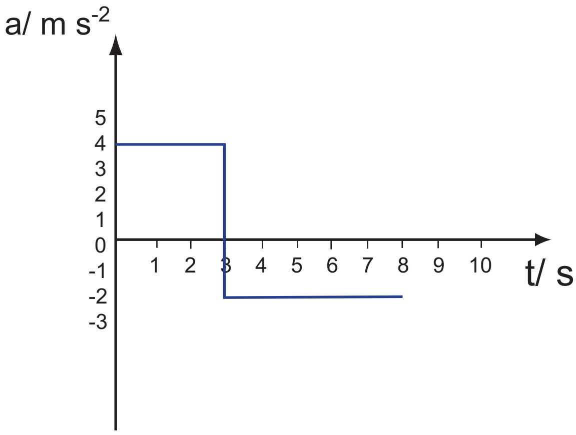
Q77: A sprinter is practising his starting technique. When his trainer fires the starting pistol, the sprinter accelerates from the starting blocks for a time t_1. He then slows down, coming to a halt after a total time t_2 . The velocity-time graph is shown below.



1. If t_1 = 2.5 s, calculate the sprinter's acceleration from t = 0 to t = t_1 , in m s $^{-2}$.
2. If t_2 = s, calculate the sprinter's acceleration from t = t_1 to t = t_2 , in m s $^{-2}$

. . . .

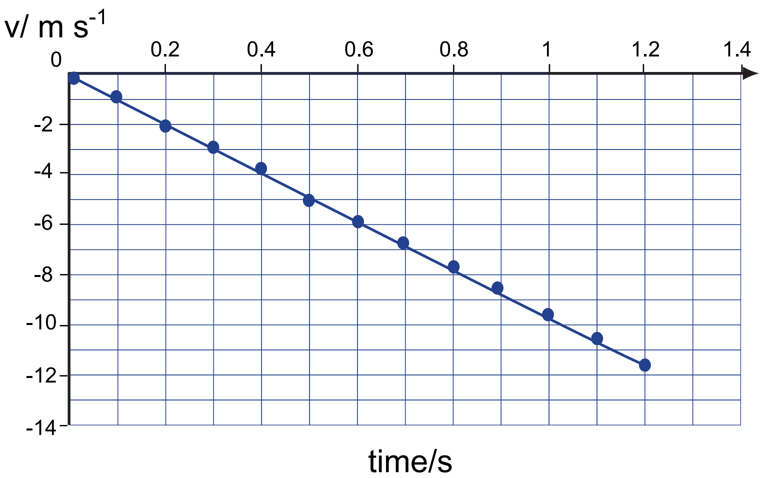
Q78: The graph below shows the acceleration against time for a car. The car is initially at rest.



1. Calculate the velocity of the car in m s $^{-1}$ after s.

. . . .

Q79: A ball is dropped near the surface of the Earth. The following graph is obtained of its motion.



What is the acceleration of the ball as it falls?

. . . .

Q80: An object is allowed to slide down a frictionless slope and its acceleration is measured. The angle is now increased.

1. What happens to the value of the acceleration?
2. Is the acceleration of the object directly proportional to the angle of the slope?

Topic 2

Forces, energy and power

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2.10 Assessment 91

Learning objective

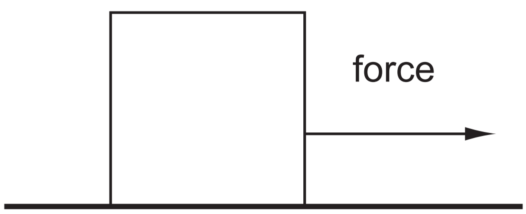
- state and apply Newton's laws of motion;
- state that friction is a force and that it always opposes the motion of an object;
- draw a velocity time graph for an object when friction is taken into account;
- describe the motion of a rocket as it burns fuel;
- use free body diagrams to analyse the forces acting on an object;
- carry out calculations involving power;
- carry out calculations involving work done, conservation of energy, kinetic energy and potential energy.

Previously we studied the displacement, velocity and acceleration of objects. Now we turn our attention to finding out what causes an object to accelerate. To study force and acceleration, we will be applying Newton's laws of motion. The technique that we use to analyse the forces acting on an object involves drawing special diagrams called free body diagrams. After focussing on moving objects (kinematics) and forces (dynamics), we will then concentrate on mechanical forms of energy: kinetic and gravitational energy. We will also examine work done which is a measure of the energy transferred during a process.

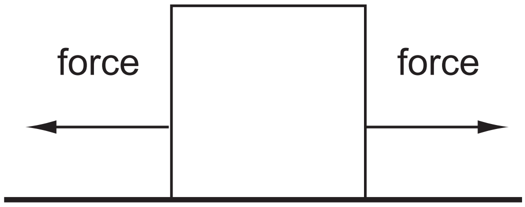
- A force is either a push or a pull. A force can change the speed of an object, the direction of an object, or the shape of an object. Force is a vector quantity and so acts in a direction. When you analyse the forces acting on an object you must remember that force is a vector.
- We will then analyse the forces acting on a body in more detail.
- We will find out how to resolve forces at angles into rectangular components so we can apply the equations we have already learned in previous topics to the movement of objects.
- We will also use free body diagrams to help analyse the motion of objects when forces are applied to them.
- The first step that you should take when trying to solve a problem involving several forces acting on an object is to sketch a free body diagram. A free body diagram is a diagram showing all the forces acting on an object. Imagine the object in isolation, so your diagram only includes that object, and then draw in all the forces that are acting on it.
- We will also see how free body diagrams are used to solve problems.
- We will examine energy and power.
- We will also look at power - the work done per second by a body.
- We also need to understand that energy cannot be created or destroyed, only changed from one form into another.

2.1 Balanced and unbalanced forces

Imagine a block sitting stationary on a frictionless surface. If a force is applied to the block it will start to move.



As this force produces a change in the motion of the object we call it an unbalanced force. An unbalanced force always causes an object to accelerate. Now imagine we apply a force equal in size but opposite in direction to the original force.



The object will now move at a constant speed. As this arrangement does not produce a change in motion in the object we describe these forces as being balanced. When the forces acting on an object are balanced the unbalanced force is zero. This is equivalent to there being no force acting on the object. Balanced forces produce no acceleration hence constant velocity.

2.1.1 Newton's laws of motion

Newton's first law of motion states that when the forces acting on an object are balanced the object will remain at rest or move with a constant velocity.

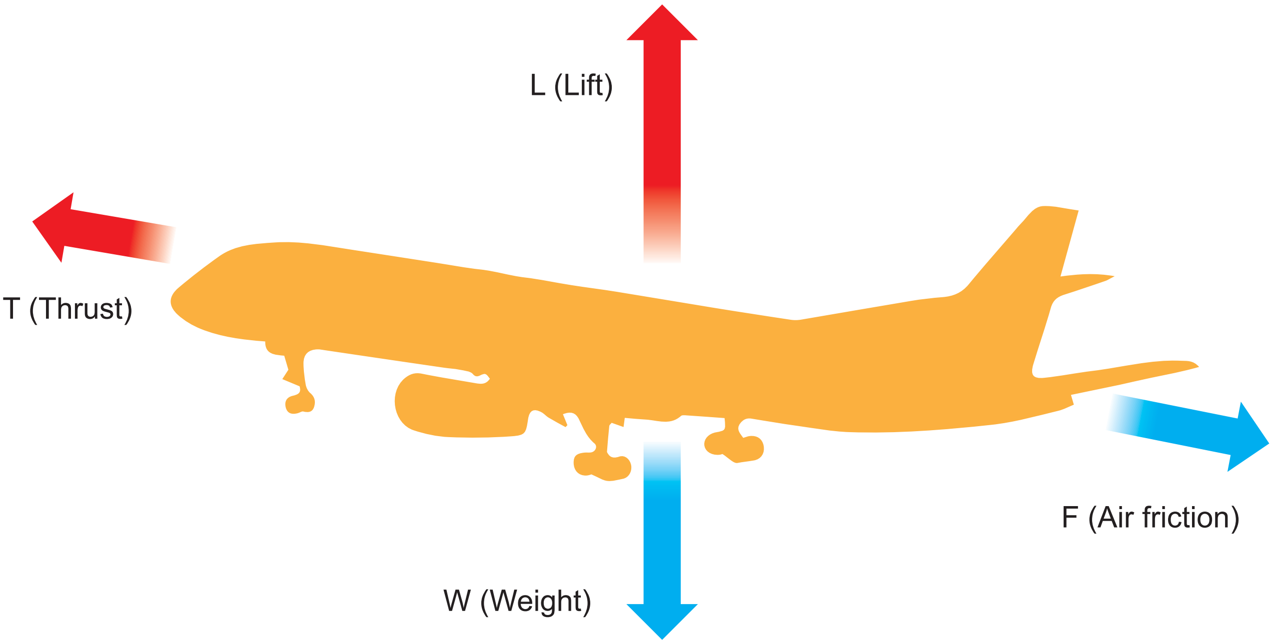
Newton's second law of motion can be stated as: "An unbalanced force acting on an object of mass *m* will cause the mass to accelerate in the direction of that force, with the acceleration proportional to the force." This law can be summed up in the equation

$$F = ma \tag{2.1}$$

The unit of force, the **newton** (N), is defined using Equation 2.1 as the force that, when applied to an object of mass 1 kg, will cause the object to accelerate at 1 m s^{-2}. From Equation 2.1, we can see that 1 N is equivalent to 1 kg m s^{-2}.
Remember that force and acceleration are both vector quantities. Newton's second law tells us that the acceleration is in the same direction as the unbalanced force.

Examples of Newton's first law

If the force of the engine and the force of friction are the same value then the forces on the car will be balanced. This means the car will not accelerate. If the car is not moving then it will remain stationary but if it is moving then it will not speed up or slow down.



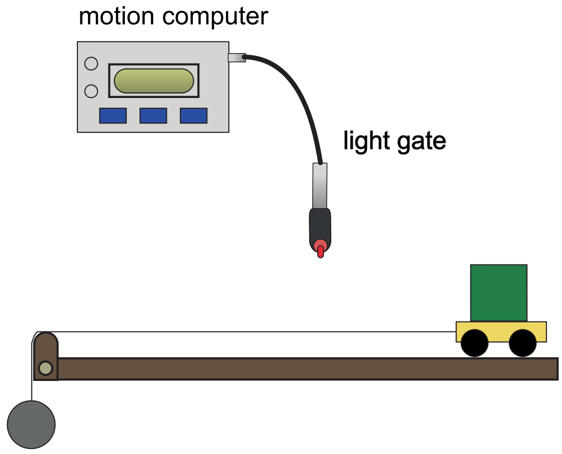
If the lift and weight are equal then the vertical forces are balanced and the plane will not accelerate upwards or downwards.

If the thrust and the air friction are of equal value then it will not accelerate horizontally.

Newton's second law Go online

There is an activity online at this stage which can be used to explore how changing the unbalanced force on an object affects its acceleration. If you do not have access to the web you should read the following explanation and make sure that you understand it.

The apparatus shown below is set up.



The vehicle sits on a linear air track. This reduces the amount of friction on the vehicle. The unbalanced force acting on the vehicle can be altered by altering the hanging mass and the acceleration is measured using a light gate and timer. When the experiment is carried out the following results are obtained.

Unbalanced Force / N	Acceleration / m s^{-2}
0	0.0
1	0.15
2	0.3
3	0.45
4	0.6
5	0.75
6	0.9
7	1.05
8	1.2
9	1.35
10	1.5

Q1: Use the data to draw a graph of these results.

. . . .

Q2: What is the relationship between the unbalanced force and the acceleration?

Example

A 4.0kg object is at rest on a smooth (frictionless) table top. A horizontal force of 24 N is applied to the object.

1. Calculate the acceleration of the object.
2. How far does the object slide along the table top in 1.0 s?

1. Using Equation 2.1,

$$F = ma$$
$$\therefore a = \frac{F}{m}$$
$$\therefore a = \frac{24}{4.0}$$
$$\therefore a = 6.0 \text{ m s}^{-2}$$

The acceleration of the object is 6.0 m s $^{-2}$

2. Now we have *a* = 6.0 m s^{-2}, *u* = 0 m s^{-1}, (initially at rest) *t* = 1.0 s and we want to find the displacement *s*. We will use the kinematic relationship $s = ut + \frac{1}{2}at^2$.

$$s = ut + \frac{1}{2}at^2$$
$$\therefore s = 0 + \left(\frac{1}{2} \times 6.0 \times 1.0^2\right)$$
$$\therefore s = 3.0 \text{ m}$$

The object slides 3.0 m along the table top in 1.0 s

Key point

Newton's third law of motion states that if one body exerts a force on a second body, then the second body will exert an equal and opposite force on the first body. So if you place a book on a desk top, the weight of the book acts downwards on the desk. The desk exerts an equal and opposite force (the *normal reaction force*) upwards on the book.

Newton's Third Law will be studied in more detail in the topic on explosions.

Quiz: Newton's second law Go online

Q3: 1 N is equivalent to

a) 1 kg m s^{-1}
b) 1 m kg^{-1} s^{-1}
c) 1 kg m s^{-2}
d) 1 kg m^{-1} s^{-2}
e) 1 m s kg^{-2}

. . . .

Q4: A 60 N force is applied to an object of mass 4.0 kg. What is the acceleration of the object?

a) 0.067 m s^{-2}
b) 15 m s^{-2}
c) 64 m s^{-2}
d) 225 m s^{-2}
e) 240 m s^{-2}

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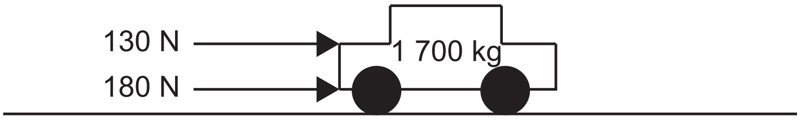
Q5: Two men are pushing a broken-down car.



The mass of the car is 1 700 kg. Calculate the acceleration of the car.

a) 0.091 m s^{-2}
b) 0.18 m s^{-2}
c) 5.5 m s^{-2}
d) 11 m s^{-2}
e) 14 m s^{-2}

. . . .

Q6: A force *F* applied to a mass *m* causes an acceleration *a*. What is the new acceleration if the force is doubled and the mass is halved?

a) *a*/4
b) *a*/2

c) *a*
d) 2*a*
e) 4*a*

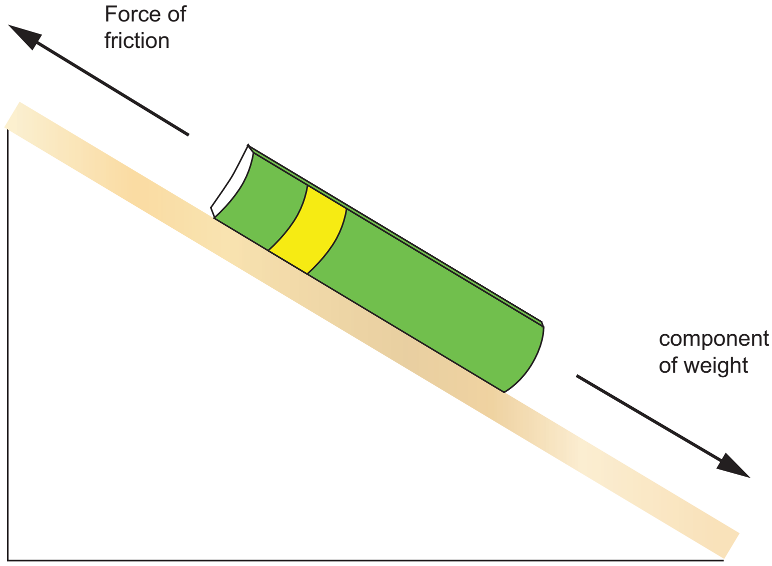
. . . .

Q7: A 50 N force is applied to a stationary object of mass 8.0 kg. What is the velocity of the object after the force has been applied for 4.0 s?

a) 0.64 m s^{-1}
b) 2.56 m s^{-1}
c) 6.25 m s^{-1}
d) 25 m s^{-1}
e) 100 m s^{-1}

2.2 Friction

Imagine you have a table with a book lying on it. When you tip up one end of the table slightly the book does not start to slide down the table. We would perhaps expect it to do so as there will be a force acting on the book. If the book does not move the forces acting on it must be balanced.

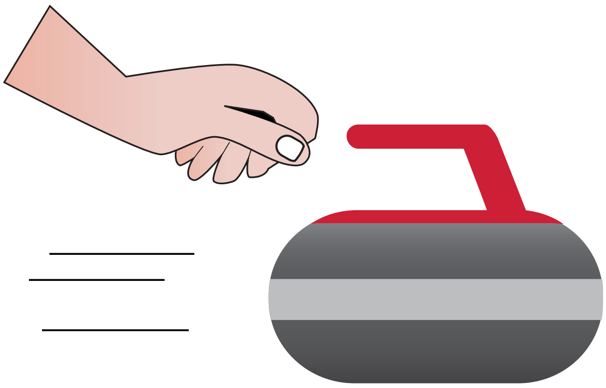


- An additional force must be present. This is the force of friction.
- The force of friction always opposes the motion of an object.
- If the object is stationary it acts to stop the object starting to move.
- If the object is already moving it acts in the opposite direction to its movement.
- If the only force acting on an object is friction it will cause the object to slow down and stop.

2.2.1 Increasing friction

You should be able to draw velocity time graphs for a moving object when no friction was present.

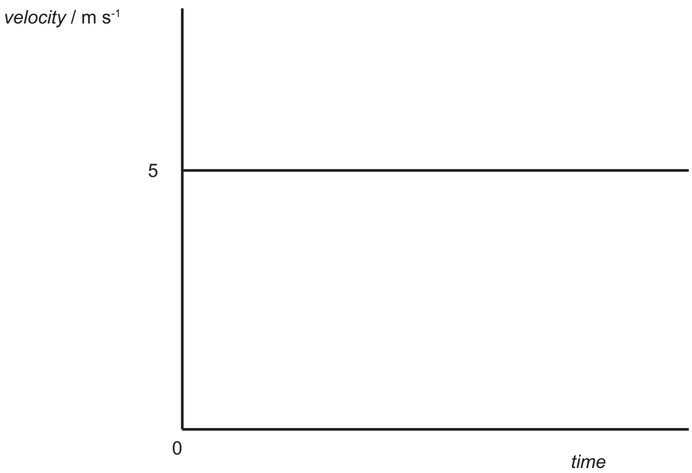
When a curling stone is slid across the ice there is virtually no force of friction acting on it.



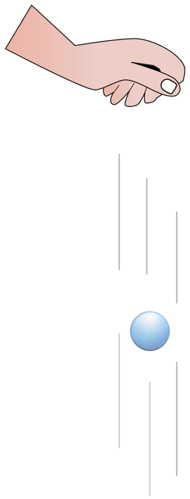
If the stone leaves the curler's hand at 5 m s^{-1} then it will continue to slide at 5 m s^{-1}.

- there is no forward force on the stone because the curler is no longer pushing it;
- there is no force opposing the motion since the ice is virtually frictionless;
- so the forces acting on the stone are balanced;
- balanced forces cause constant velocity.

This is shown in the following velocity-time graph.



When a small steel ball is dropped from rest it accelerates due to its weight.
The force of friction /air resistance acting on a small steel ball, falling less than 2 metres, is very small and can be ignored.

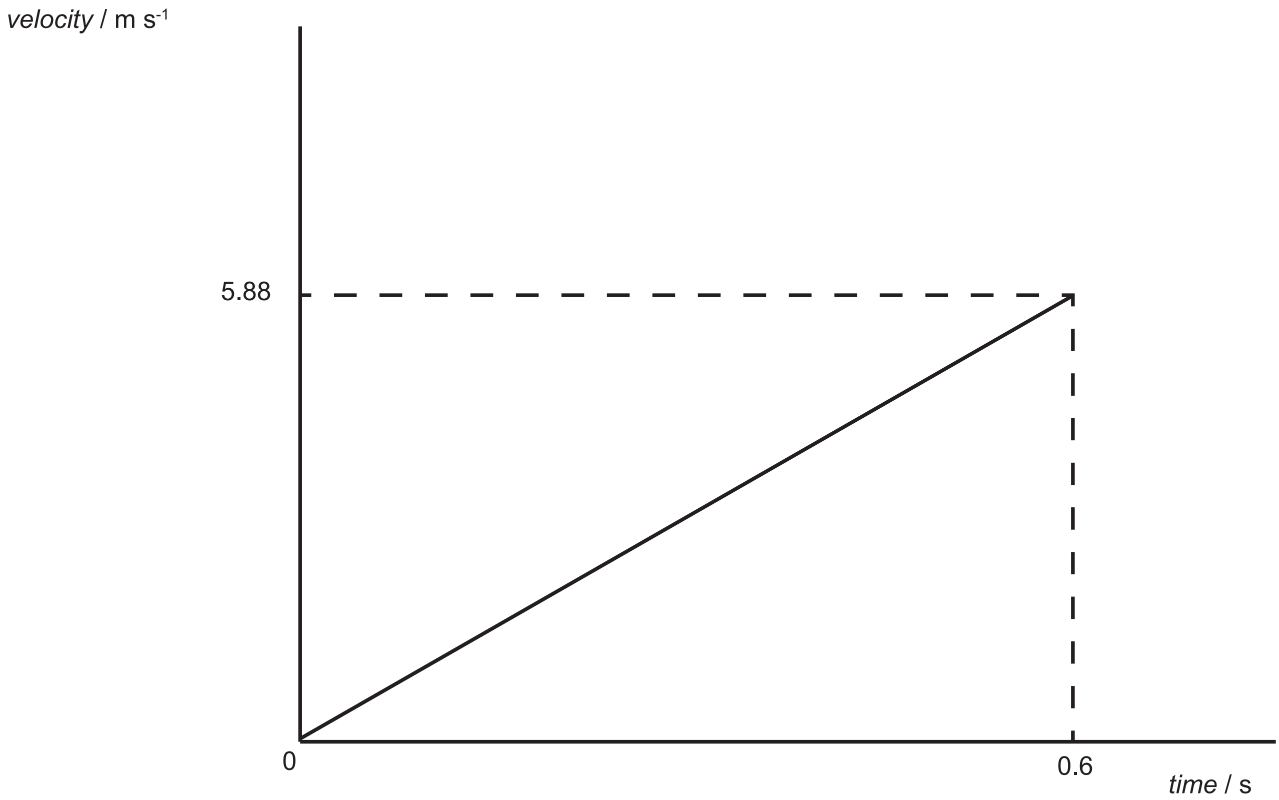


When the steel ball is dropped, its initial velocity is 0 m s^{-1}.
It will accelerate at 9.8 m s^{-2}.

When the steel ball is dropped, its initial velocity is 0 m s^{-1}:

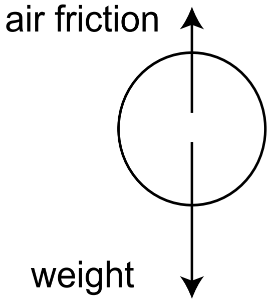
- It will accelerate at 9.8 m s^{-2};
- there is no force opposing the motion since there is virtually no force of friction/air resistance;
- so the forces acting on the ball are unbalanced;
- an unbalanced force causes an acceleration.

The velocity of the steel ball is represented in the following velocity-time graph.

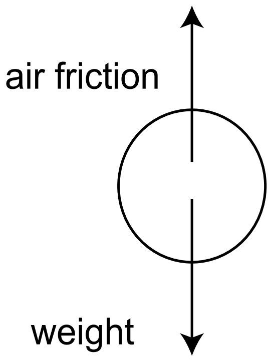


We will now look at an example that takes friction into account:

Example If an object is dropped from a height it will start to accelerate towards the ground. The force causing it to accelerate is caused by the object's weight. As the object falls it experiences a force due to air friction. Remember that friction always acts in the opposite direction to the direction the object is travelling.



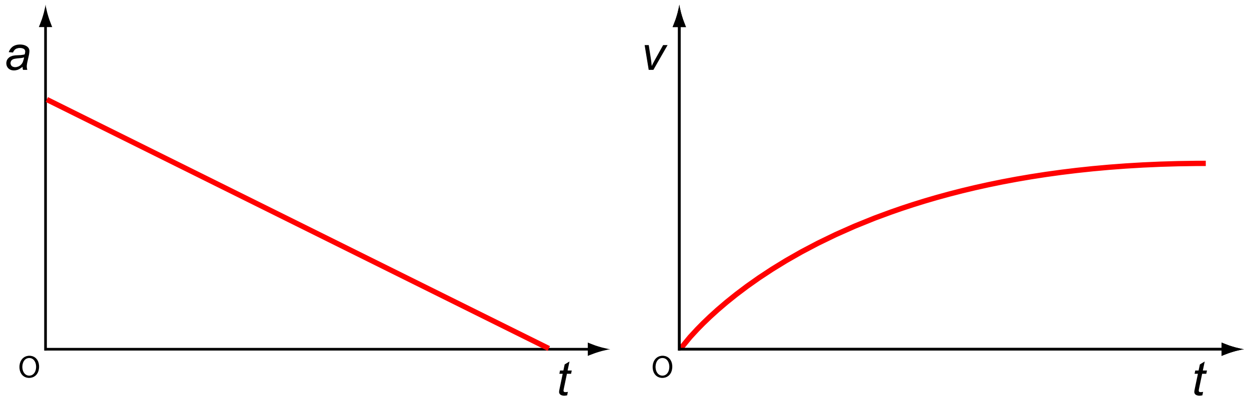
We can see that the accelerating (resultant) force is now less than the weight so the acceleration of the object has decreased. Note the object is still speeding up but the rate at which its speed is increasing is now less than it was before. As the object travels faster it passes through a larger volume of air each second so the air friction acting on it increases as it falls. At some stage, as long as it is falling for long enough, the magnitude of the friction force will equal the weight of the object. As these two forces are in opposite directions they will cancel each other out and the unbalanced force will be zero. The forces acting on the object are balanced.



As the forces acting on the object are balanced it obeys Newton's first law of motion. The acceleration of the object will now be zero and so it falls at a constant velocity. This is called the **terminal velocity of the object**.

This pair of graphs describes the situation when a skydiver jumps out of a plane. The acceleration is greatest at the start of the jump but gradually decreases as the frictional forces increase.

Figure 2.1: Acceleration-time and velocity time graphs for a skydiver



Notice that as the acceleration decreases, so too does the slope of the velocity-time graph resulting in a curve that ends up as a horizontal line.

When the skydiver reaches this point her velocity is constant. This is her terminal velocity.

At this point the forces acting on her must be balanced (see Newton's first law) and so the force of friction must be equal to her weight.

2.2.2 Rain drops

Let us consider a raindrop falling from a height of 1000 m.

We can use the equation $v^2 = u^2 + 2as$ to calculate what its velocity would be if there was no air friction acting on it.

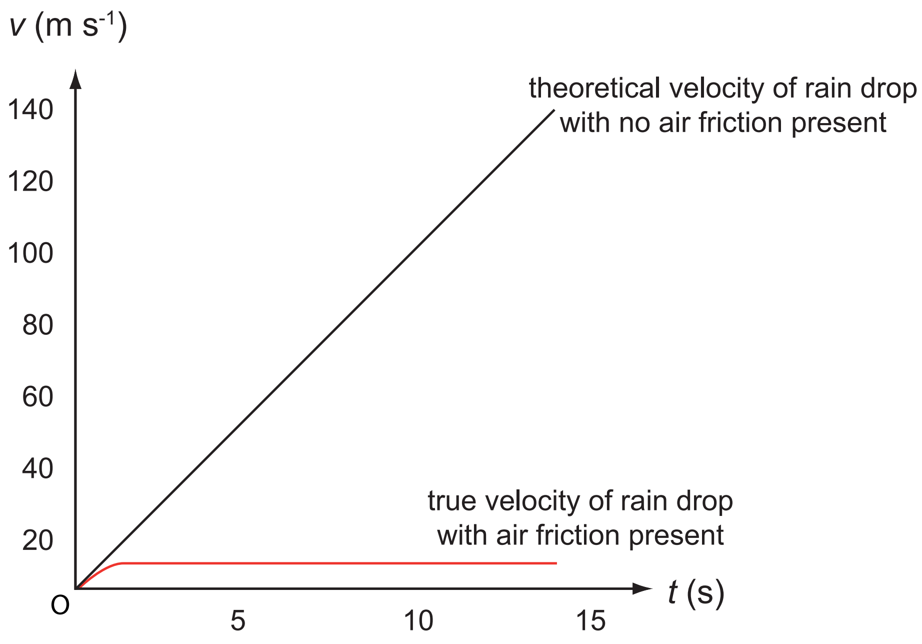
$v^2 = 0^2 + 2 \times 9.8 \times 1000$
$v^2 = 19600$
$v = 140 \text{ m s}^{-1}$

That is a velocity of over 500 kilometres per hour!

Raindrops are travelling much slower than that when they hit the ground as they reach their terminal velocity before they are close to the ground. The terminal velocity of raindrops is typically less than 10 m s^{-1} (20 miles per hour).

The following velocity time graph shows how the velocity of a rain drop varies with and without air friction.

Figure 2.2: Velocity time graph comparing velocity of a rain drop with air friction present and not present

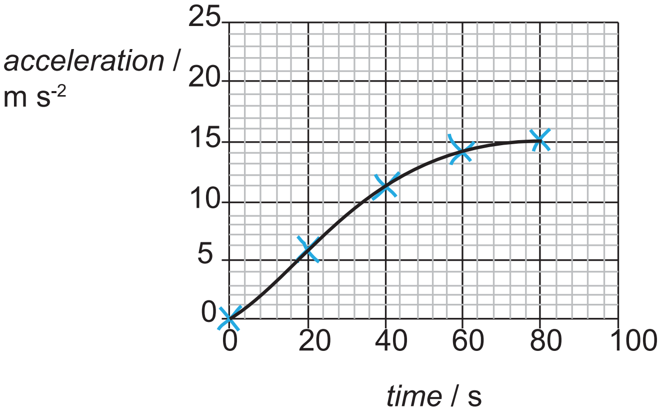


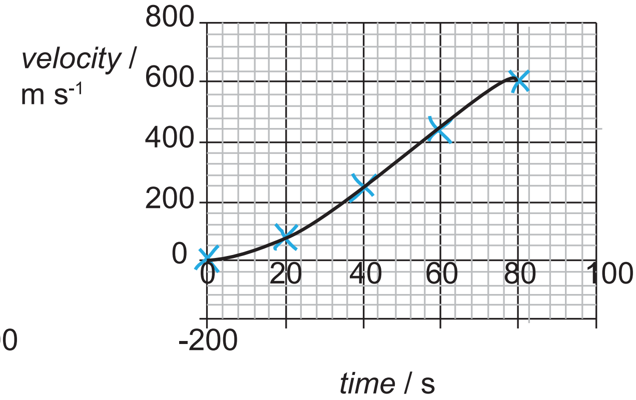
2.2.3 Space rockets

A space rocket produces a constant thrust. At first glance this would appear to mean that the rocket will have a constant acceleration but this is not the case.

As the rocket moves its engines burn fuel. This means the mass of the rocket becomes less. As a result the acceleration of the rocket will increase. Acceleration-time and velocity-time graphs are shown below.

Figure 2.3:





These graphs show that the acceleration in the first 80s of flight is not constant. The acceleration increases from 2.5 m s-2 to 20 m s-2 during the first 80s of flight. This can be explained as follows:

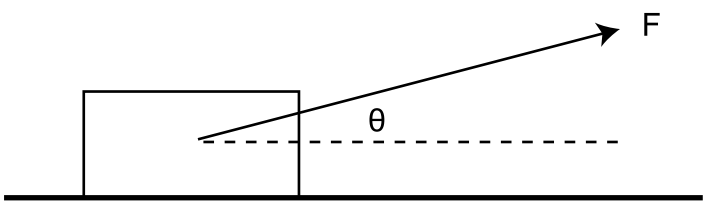
- Thrust of engines is constant;
- Mass decreases due to the fuel being used by rocket;

- Weight of rocket = mass of rocket x gravitational field strength decreases (due to decrease in mass);
- The unbalanced force, $F_{unbalanced}$ = Thrust of engines - weight of rocket $\Rightarrow$ $F_{unbalanced}$ must increase;
- Since Funbalanced = $m \times a \Rightarrow a = \frac{F_{unbalanced}}{m}$ $\Rightarrow$ a must increase

The increase in acceleration means that the velocity increase in each 20 second interval also increases. This is why the slope of the velocity-time graph increases.

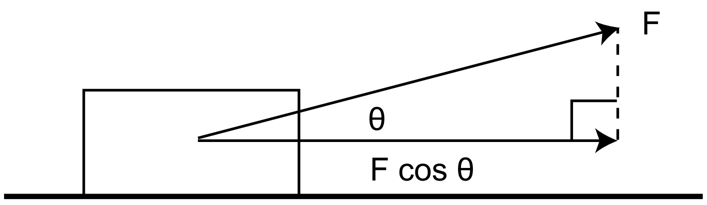
2.3 Resolving forces into rectangular components

Consider a box being pulled along a frictionless surface by a force at an angle.



The box will accelerate along the surface.

To work out the size of the accelerating force we will have to work out the size of the horizontal component of the force. To do this we use trigonometry.



- We can see from the diagram that the horizontal component of this force is equal to F cos θ. (In this context the word *component* means *part of*.)
- We could find the vertical component by a similar method.
- The vertical component will be equal to F sin θ.
- These two components of a vector are called the rectangular components of the vector because they are at right angles to each other.
  Knowing the vertical and horizontal components of a vector can be very useful when analysing the motion of an object.

2.4 Using free body diagrams

A free body diagram shows the forces (sometimes the components of the forces) acting on an object.

When an object is in equilibrium, there is no unbalanced force acting in a particular direction, and the acceleration of the object is zero. If we analyse the components of all the forces acting on the object, their vector sum *in any direction* will be zero. We often look at the rectangular components of all the forces, such as the components acting parallel and perpendicular to a surface.

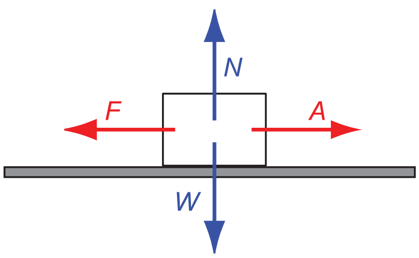
Example A portable television of mass 5.0 kg rests on a horizontal table. A force of 65 N applied horizontally to the television causes it to slide at a constant velocity across the table top.

Calculate:

1. the normal reaction force which the table exerts on the television;
2. the (horizontal) frictional force acting on the television.

Answer:
There are four forces acting on the television. Its weight W acts downwards, the normal reaction force N acts upwards, the applied force A acts horizontally, and the frictional force F acts horizontally in the opposite direction to A. We can show all these forces on a free body diagram such as Figure 2.4

Figure 2.4: Free body diagram



1. Vertically (perpendicular to the surface), the television is in equilibrium, since it is not moving in a vertical direction, so the forces acting upwards equal the forces acting downwards. Hence

   $N = W$

   $\therefore N = m \times g$

   $\therefore N = 5.0 \times 9.8$

   $\therefore N = 49N$

   The vertical reaction force is therefore 49 N upwards.

2. Horizontally (parallel to the surface), the television is also in equilibrium since it is moving with constant velocity, so the resultant horizontal force must be zero.

   $F = A$

   $\therefore F = 65 \text{ N}$

   The frictional force is therefore 65 N to the left.

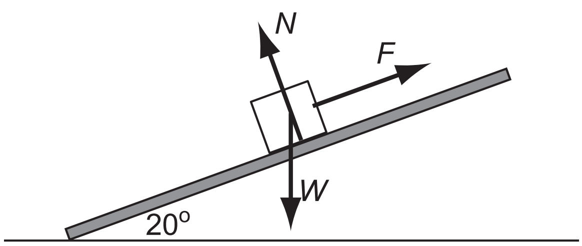
That example was probably straightforward enough for you to solve without needing a diagram, but a free body diagram can prove invaluable in more complex situations.

Example

Suppose the table is now lifted at one end, so that the 5.0 kg television is now resting on a surface inclined at 20° to the horizontal. The only forces acting on the television are its weight, the normal reaction force of the table top, and the frictional force acting parallel to the slope. If the television is stationary, calculate the magnitude of the frictional force.

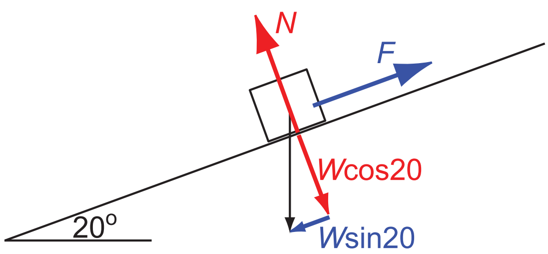
Answer:
Let us label the three forces as *W* for the weight, *N* for the normal reaction force and *F* for the frictional force, which acts up the slope to prevent the television sliding down the slope. These forces are shown in Figure 2.5

Figure 2.5: Television resting on a tilted table



To solve this problem, we will consider the components of *W*, *N* and *F* acting parallel and perpendicular to the slope. Whilst *N* has no component parallel to the slope and *F* has no component perpendicular to the slope, *W* acts at an angle to the slope. The free body diagram (Figure 2.6) should clarify the situation.

Figure 2.6: Free body diagram of the television on a tilted table



Make sure that you understand why the component of *W* acting parallel to the slope is *W*sin20 and the component acting perpendicular to the slope is *W*cos20. We can now calculate *F* by analysing the components acting parallel to the table top.

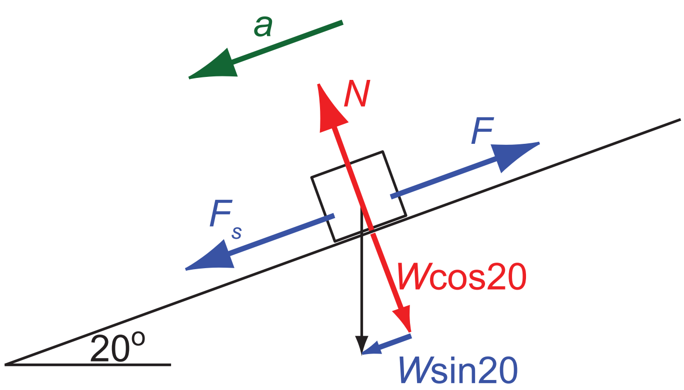
$$\text{component of weight parallel to slope} = W \sin 20$$
$$\therefore \text{component of weight parallel to slope} = mg \sin 20$$
$$\therefore \text{component of weight parallel to slope} = 5.0 \times 9.8 \times 0.342$$
$$\therefore \text{component of weight parallel to slope} = 17 \text{ N}$$

Since the television is stationary the forces parallel to the slope must be balanced so the frictional force up slope, F, must also = 17 N.

2.5 Objects undergoing acceleration

If an object is accelerating, there must be an unbalanced force acting on it in the direction of the acceleration. Let us continue with the example of the television (mass 5.0 kg) resting on a tilted table. The component of weight acting down the slope, parallel to the slope, is often labelled as F_s (F_{slope}). If the frictional force *F* between the television and the table top is only 8.0 N, then an analysis of the forces acting parallel to the slope show an unbalanced force acting down the slope. Again, we use a free body diagram as shown in Figure 2.7.

Figure 2.7: Free body diagram of the television sliding down the slope



The component of weight down the slope F_s can be found as follows.

- F_s = W$\sin\theta$ where θ is the angle between the slope and the horizontal.
- F_s = mg $\sin\theta$ try to remember this relationship as it is not on the list provided.

We have an acceleration *a* acting down the slope. The free body diagram shows us that the unbalanced force acting in this direction is *W sin 20 - F*. So in this case, from Newton's second law:

W sin 20 - F = ma

We can solve this equation to find the acceleration *a*:

$$W \sin 20 - F = ma$$
$$\therefore a = \frac{W \sin 20 - F}{m}$$
$$\therefore a = \frac{5.0 \times 9.8 \times 0.342 - 8.0}{5.0}$$
$$\therefore a = \frac{8.758}{5.0}$$
$$\therefore a = 1.8 \text{ m s}^{-2}$$

The acceleration down the slope is therefore 1.8 m s^{-2}.

So we have used a free body diagram along with Newton's second law to determine the acceleration. We can apply this technique to solve any problem in this topic.

Examples

1. Lifts

If you stand on a set of bathroom scales, the scales read the reaction force to your weight.

For example if you have a mass of 50 kg, you weight 50 $\times$ 9.8 = 490 N.

When you stand on a set of scales you exert a force of 490 N on them, so according to Newton's third law of motion they exert a force of 490 N back on you. The scales read this reaction force (scales are normally calibrated to read in kilogram s but they are actually measuring force).

If you stand on a set of bathroom scales in a lift that is either stationary or moving with a constant speed the scales will still read 490 N.

However if the lift is accelerating upwards then the reaction force of the scales will be greater.

For example if the lift is accelerating upwards at 1.0 m s^{-2} then you must be accelerating at the same rate. This means that the lift must exert a force on you of 50 N. The lift's reaction force to your weight is already 490 N so the total force on you by the lift is now 540 N. This is the reading that the scales would now give.

The opposite is true if you were accelerating downwards at 1.0 m s^{-2}. The lift would now exert 50 N less force on you so the scale would read 440 N.

If the lift cable was to be cut and the lift allowed to fall both the lift and you would be falling freely so the scale would read 0 N. You would apparently be weightless.

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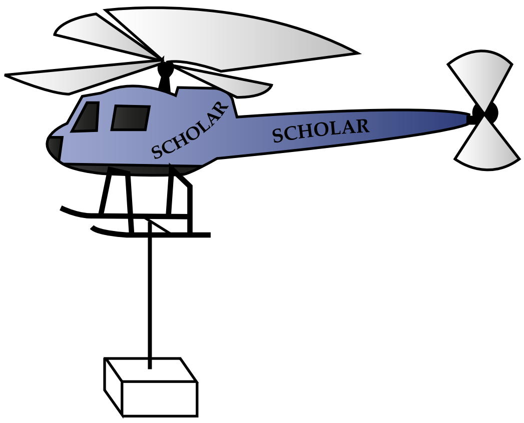
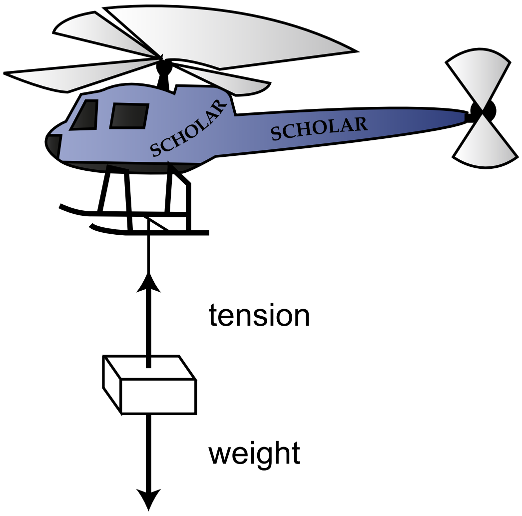
2. Tension

When a helicopter carrying a crate hovers above the ground without moving the forces acting on the crate must be balanced.



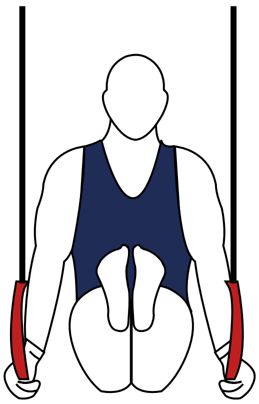
The weight of the crate acts downwards so there must be a force in the cable that stops it accelerating downwards. This force is the tension.



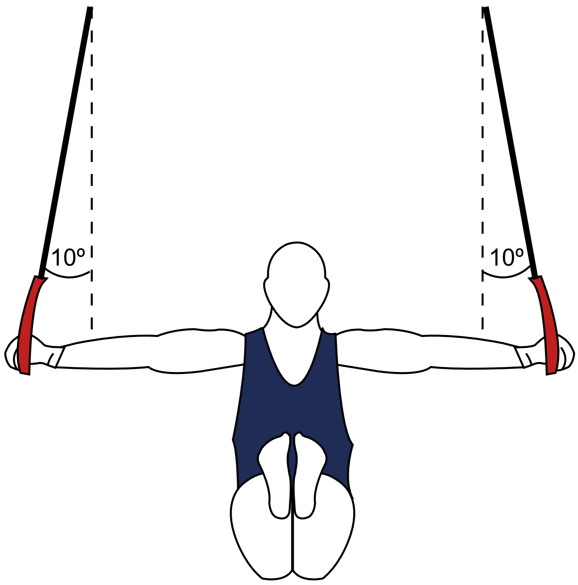
This was a very simple example. Let us now look at a more complex situation.

Example : Gymnast on rings

A gymnast is using the rings apparatus. When he hangs straight down from the rings as shown each rope supports half his weight so the tension in each rope is equivalent to half his weight but in an upwards direction.

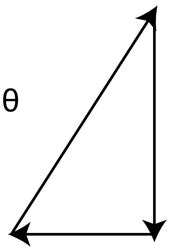


If he now stretches his arm out sideways the tension in each rope increases.



Because he is not moving the forces must be balanced. A diagram helps illustrate how the tension must increase.

tension = ½ weight ÷ cos θ



downwards force = ½ weight

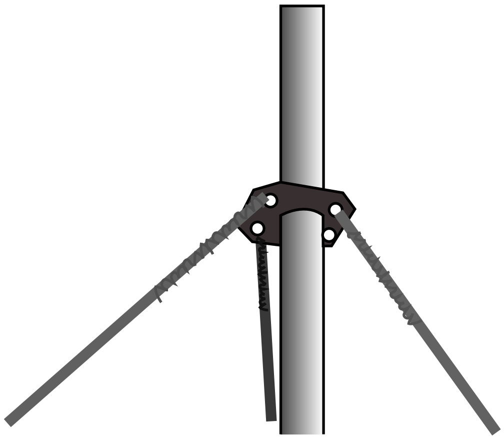
We can see from the diagram that tension is now = $^1/_2$ weight $\div$ cos θ

When an object is held stationary by the forces acting on it, it is said to be ***in equilibrium***

An example of a structure in equilibrium is a mast aerial.

Examples

1. Mast aerial

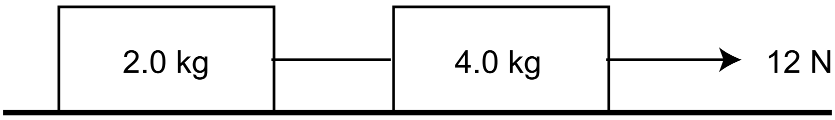


We can see from this close up of the mast that it is held in position by three guy cables. If the mast starts to move in any direction there will be a component of tension in the opposite direction from at least one of the cables. This will tend to pull the mast back to its original position.

....

2. Two blocks

Let us now think about two blocks on a frictionless surface joined together by a string.



An unbalanced force of 12 N is applied to the blocks. What will be the tension in the string? First we should calculate the acceleration of the whole system.

$a = {}^{F}/_{m}$
$a = {}^{12}/_{6}$
$a = 2\ \text{m s}^{-2}$

So the whole system is accelerating at $2\ \text{m s}^{-2}$.

The accelerating force on the 2 kg block is being provided by the tension of the string. We can now work this out.

$F = ma$
$F = 2 \times 2$
$F = 4\ \text{N}$

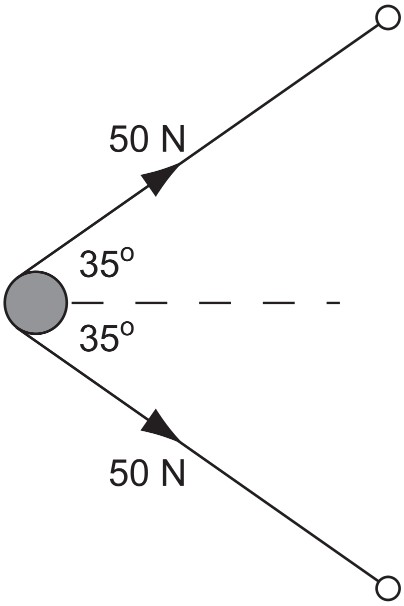
Therefore the tension in this string is 4 N.

. . . .

3. Catapult

Figure 2.8 shows a catapult, just about to launch a pebble.

Figure 2.8: The catapult just before the pebble is released

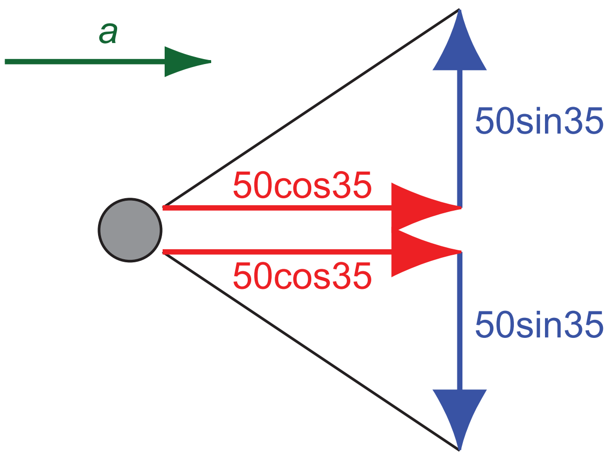


The mass of the pebble is 100 gram s. The tension in the catapult elastic is 50 N. At the instant the pebble is released, what is

1. the force acting on the pebble?
2. the acceleration of the pebble?

Answer:
We start by drawing a free body diagram.

Figure 2.9: Free body diagram for the catapult



1. As we have drawn it in Figure 2.9, there is an unbalanced force acting to the right, causing an acceleration *a*. By taking the components acting in this direction, we can calculate the unbalanced force *F*.

   $F = 50 \cos 35 + 50 \cos 35$
   $\therefore F = 2 \times 50 \times 0.819$
   $\therefore F = 82$ N
   The unbalanced force is therefore 82 N to the right.
2. Now we have a value for the unbalanced force, we can use $F = ma$ to find the acceleration.

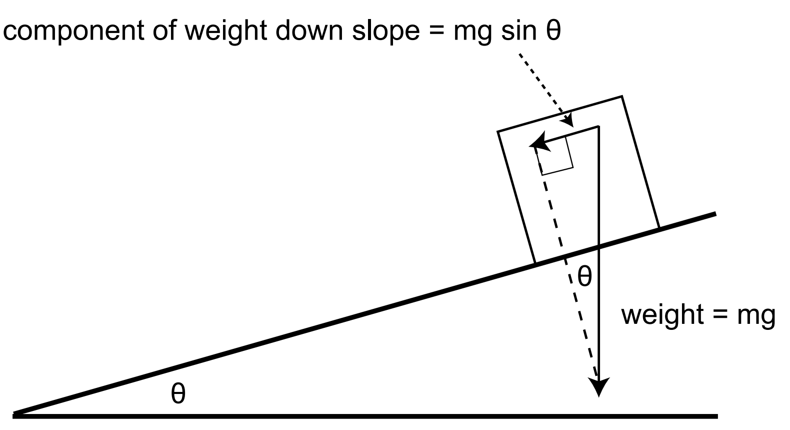
   $F = ma$
   $\therefore a = {}^{F}/_{m}$
   $\therefore a = {}^{82}/_{0.1}$
   $\therefore a = 820 \text{ m s}^{-2}$
   The acceleration of the pebble is 820 m s^{-2} to the right.

2.5.1 Acceleration of an object

The following two activities should help to explain the theory.

Mass on a slope Go online

Consider two objects resting on two different frictionless slopes. A 5 kg mass is sitting on a slope that makes an angle of 20° to the horizontal. A different mass of 10 kg is sitting on a slope that makes an angle of 10° to the horizontal. Which of these two masses will have the greatest acceleration? We start by drawing a free body diagram.



We can see that the component of force acting down the slope = mg sin θ.

Consider the 5 kg mass first.

F = mg sin q = 5 $\times$ 9.8 $\times$ sin 20 = 16.8 N

So the acceleration of the block is

$a = {}^{F}/_{m} = {}^{16.8} / {}^{5} = 3.36 \text{ m s}^{-2}$

Now consider the 10 kg mass.

$F = mg \sin q = 10 \times 9.8 \times \sin 10 = 17.1$ N

So the acceleration of the block is

$a = {}^{F}/_{m} = {}^{17.1}/_{10} = 1.71$ m s^{-2}

Therefore the 5 kg block has the greatest acceleration.

Now try this question.

If the mass is 4.0 kg and the slope angle is 35°. What is the acceleration of the mass down the slope?

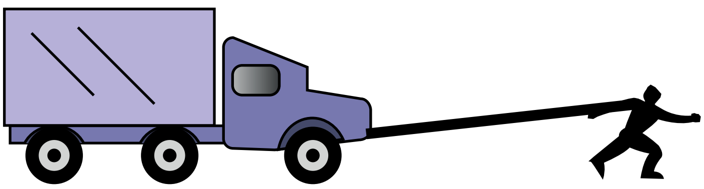
Using a free body diagram, the forces acting on an object can be analysed to calculate its acceleration.

World's strongest man Go online

At this stage there is an online activity. If however you do not have access to the internet you should read the explanation which follows and make sure that you understand it.

If an unbalanced force is acting on an object the object will accelerate.

Let us think about a 'world's strongest man' truck pulling completion. Each truck has a mass of 8000 kg and has to be pulled through a distance of 50 m. The three contestants pull a truck using a harness around their shoulders.



The contestants are described below.

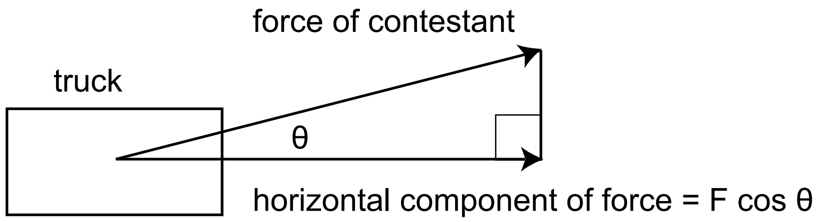
- Stige pulls with a force of 3200 N at an angle of 20° to the horizontal.
- Issac pulls with a force of 3400 N at an angle of 25° to the horizontal.
- Vlad pulls with a force of 3700 N at an angle of 35° to the horizontal.

The frictional forces are the same for each contestant.

Who will win the contest?

We can calculate which of the contestants will by working out which exerts the largest horizontal force on the truck. The contestant with the largest resultant horizontal force will have the largest acceleration and hence win the contest.

We will draw a free body diagram.



From this we can calculate the horizontal force that each contestant exerts on the truck. The horizontal component of force = F cos θ

- Stige = 3200 $\times$ cos 20 = 3010 N
- Issac = 3400 $\times$ cos 25 = 3080 N
- Vlad = 3700 $\times$ cos 35 = 3030 N

As Issac exerts the biggest horizontal force he will have the largest acceleration and hence will win the contest.

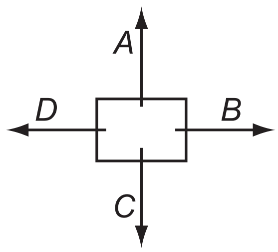
The acceleration of an object can be calculated using free body diagrams and applying Newton's second law.

Quiz: Free body diagrams Go online

Useful data:

acceleration due to gravity *g*	9.8 m s^{-2}

Q8: Four forces *A*, *B*, *C* and *D* act on an object as shown.



The magnitudes of the forces are: *A* = 10 N, *B* = 20 N, *C* = 10 N and *D* = 40 N. In which direction is the object accelerating?

a) In the direction of *A*
b) In the direction of *B*
c) In the direction of *C*
d) In the direction of *D*
e) The object is in equilibrium and is not accelerating.

. . . .

Q9: A 15 kg crate is sliding down a slope at constant velocity. If the slope is inclined at 25° to the horizontal, calculate the magnitude of the frictional force acting on the crate.

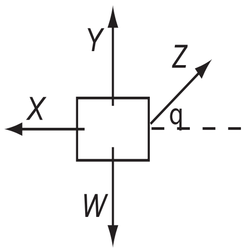
a) 62 N
b) 69 N
c) 133 N
d) 147 N
e) 3700 N

. . . .

Q10: On a building site, a 5.0 kg block is being lowered on the end of a rope. If the downward acceleration of the block is 1.6 m s^{-2}, what is the force of the block on the rope?

a) 8.0 N
b) 16 N
c) 41 N
d) 47 N
e) 57 N

The following diagram relates to the next two questions.



Q11: Four forces *W*, *X*, *Y* and *Z* act on an object. *X* has magnitude 35 N. *Z* has magnitude 40 N, and acts at an angle θ = 40°. What is the magnitude and direction of the resultant force in the horizontal direction?

a) 5.0 N ⇒
b) 3.8 N ⇐
c) 1.4 N ⇒
d) 4.4 N ⇐
e) 9.3 N ⇒

. . . .

Q12: Referring to the diagram in the previous question, the magnitude of *W* is 60 N. If the unbalanced force is zero in the direction of *W*, what is the magnitude of *Y*?

a) 0 N
b) 13 N
c) 20 N
d) 29 N
e) 34 N

2.6 Energy

Work is done whenever a force is applied to move an object through any distance. The energy E_w used in applying a force F over a distance d can be summed up in the equation

$$\text{work done} = \text{force} \times \text{distance}$$
$$E_w = F \times d \tag{2.2}$$

Energy (or work done) is a scalar quantity, and is measured in joules J.

The **kinetic energy** of an object is the energy that the object possesses due to its motion. An object of mass m moving with speed v has kinetic energy given by

$$E_k = \frac{1}{2}mv^2 \tag{2.3}$$

We can think of kinetic energy as the work that would have to be done to bring the object to rest.

The other type of energy we must consider is **potential energy**. This is energy stored in an object due its position, its shape or its state. For example, if you stretch an elastic band, potential energy is stored in the band. When you release the band, that energy is used to restore the band to its natural shape - snapping your fingers in the process! If you lift a book up from the floor and place it on a table, you have increased the gravitational potential energy of the book. This potential energy is converted into kinetic energy if you nudge the book off the table and it falls back to the floor.

This conversion from potential energy to kinetic energy is an example of the conservation of energy. Energy can be changed from one type to another but it cannot be created or destroyed. The total energy before an action must equal the total energy after the action.

If you raise an object of mass m through a vertical height h, the increase in the potential energy of the object is given by Equation 2.4.

$$E_p = mgh \tag{2.4}$$

g is the gravitational field strength. We can apply the principle of **conservation of energy** to solve some problems involving vertical motion.

Example : Toy rocket

A toy rocket is projected vertically upwards from the ground, with an initial velocity of 30 m s^{-1}. Show that the maximum height reached by the rocket is 46 m by using

1. kinematic relationships;
2. conservation of energy.

Answer:

1. Using the kinematic relationships, we must find the displacement of the rocket at the instant when its velocity is momentarily zero, just before it starts to fall back to Earth. We have an initial (upwards) velocity *u* = 30 m s^{-1}, final velocity *v* = 0 m s^{-1} and acceleration *a* = -*g* m s^{-2}, so we will use the kinematic relationship $v^2 = u^2 + 2as$.

$$v^2 = u^2 + 2as$$
$$\therefore s = \frac{v^2 - u^2}{2a}$$
$$\therefore s = \frac{0^2 - 30^2}{2 \times (-9.8)}$$
$$\therefore s = 46 \text{ m}$$

The displacement of the rocket is 46 m.

2. At the instant the rocket is projected, its potential energy is zero, and all the energy is kinetic energy. At its maximum height *h*, when the velocity is momentarily zero, all the kinetic energy is converted to potential energy.

$E_{k\text{ at ground}} = E_{p\text{ at h}}$

$$\frac{1}{2}mv^2 = mgh$$
$$\therefore \frac{1}{2}v^2 = gh$$
$$\therefore h = \frac{v^2}{2g}$$
$$\therefore h = \frac{30^2}{2 \times 9.8}$$
$$\therefore h = 46 \text{ m}$$

This method also gives a displacement for the rocket of 46 m.

Projectile motion Go online

There is an online activity at this stage which can be used to examine the gravitational and kinetic energy of a projectile as it flies through the air.

If you do not have access to the internet read the following explanation and make sure that you understand it.

If we consider a 2.0 kg ball projected horizontally with a speed of 5.0 m s^{-1} from a cliff that is 10 m high.

The total energy of the ball is its potential energy plus its kinetic energy.

$E = mgh + {}^1/_2\, mv^2$
$E = (2 \times 9.8 \times 10) + (0.5 \times 2 \times 52)$
$E = (2 \times 9.8 \times 10) + (0.5 \times 2 \times 25)$
E = 196 + 25 E = 221 J

As the object falls its potential energy converts to kinetic energy. This means that just before it strikes the ground it will have 0 J of potential energy and 221 J of kinetic energy.

At any point during this projectile's motion the total of its kinetic energy and potential energy will be equal to 221 J.

The total energy (the sum of kinetic and potential energies) of a projectile is constant.

2.7 Power

Power is defined as the rate at which work is being done.

$$P = \frac{E}{t} \tag{2.5}$$

Power is measured in watts (W), where 1 W is equivalent to 1 J s^{-1}. If a total of 2000 J of work is done over a period of 40 s, then the average power over that period of time is 50 W.

Examples

1. Ball

A ball of mass 2.0 kg is held 4.0 m above the ground.

1. Calculate the potential energy of the ball.
2. The ball is now released. Assuming no loss due to friction calculate the speed of the ball

just before it hits the ground.

3. The ball loses 18.4 J of energy when it makes contact with the ground. Calculate the rebound height of the ball.

Answer:

1. E_p = mgh
   $E_p = 2 \times 9.8 \times 4$
   E_p = 78.4 J
   The potential energy of the ball is 78.4 J.
2. $E_k = {}^1/_2\ mv^2$
   $v^2 = 2 \times {}^{Ek}/_{m}$
   $v^2 = 2 \times {}^{78.4}/_{2}$
   v^2 = 78.4
   v = 8.9 m s^{-1}
   The speed of the ball just before it reaches the ground is 8.9 m s^{-1}.
3. E_p = 78.4 - 18.4 = 60 J
   E_p = mgh
   $h = {}^{Ep}/_{(mg)}$
   $h = 60 / (2 \times 9.8)$
   $h = {}^{60}/_{19.6}$
   h = 3.1 m
   The ball rebounds to a height of 3.1 m.

. . . .

2. Motorcycle and rider

The total mass of a motorcycle and rider is 250 kg. During braking, they are brought to rest from a speed of 16.0 m s^{-1} in a time of 10.0 s.

1. Calculate the maximum energy which could be converted to heat in the brakes.
2. Calculate the maximum average power developed by the brakes

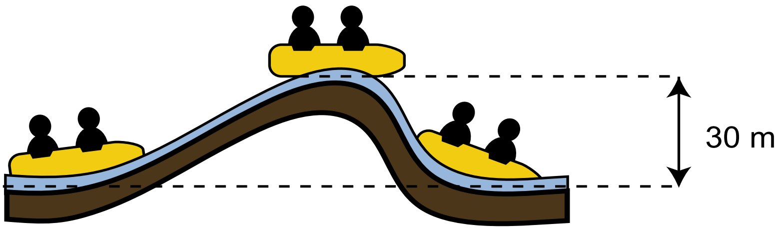
Answer:

1. $E_k = {}^1/_2\ mv^2$
   $E_k = {}^1/_2 \times 250 \times 16^2$
   $E_k = {}^1/_2 \times 250 \times 256$
   E_k = 32000 J
   The maximum amount of energy that can be converted into heat is 32 kJ.
2. $P = {}^E/_t$
   $P = {}^{32000}/_{10}$
   P = 3200 W
   The maximum average power developed by the brakes is 3200 W.

. . . .

3. Theme park

A theme park has a water splash ride. A carriage loaded with passengers is raised through a height of 30 m to the top of the ride. The combined mass of the carriage and the passengers is 1400 kg.



1. Calculate the gain in gravitational potential energy of the carriage and passengers when it is taken to the top of the ride.
2. A winch takes 20 s to pull the carriage to the top of the ride. Calculate the minimum average power of the winch.
3. If 132000 J of energy are lost to friction as the carriage slides down the slope what is the kinetic energy at the bottom of the slope?
4. What is the speed of the carriage at the bottom of the slope?

Answer:

1. $E_p = mgh$
   $E_p = 1400 \times 9.8 \times 30$
   $E_p = 411600$ J
   Remember to round off final answers.
   $E_p = 412000$ J
   The gain in potential energy is 412000 J.
2. $P = {}^{E}/_{t}$
   $P = {}^{412000}/_{20}$
   $P = 20600$ W
   The minimum power required is 20 600 W.
3. $E_k = 412000 - 132000 = 280000$ J
   The kinetic energy at the bottom of the slope is 280000 J.
4. $Ek = {}^{1}/_{2}\, mv^2$
   $v^2 = 2\, {}^{Ek}/_{m}$
   $v^2 = 2 \times {}^{280000}/_{1400}$
   $v^2 = 400$
   $v = 20$ m s^{-1}
   The speed of the carriage at the bottom of the slope is 20 m s^{-1}.

Quiz: Energy and power Go online

Useful data:

acceleration due to gravity *g*	9.8 m s^{-2}

Q13: A 5.0 kg object is travelling with a uniform velocity of 12 m s^{-1}. What is the kinetic energy of the object?

a) 30 J
b) 150 J
c) 180 J
d) 360 J
e) 1800 J

. . . .

Q14: A roof tile drops 10 m from the top of a building onto the ground. Calculate the speed of the tile just before it hits the ground.

a) 9.9 m s^{-1}
b) 14 m s^{-1}
c) 44 m s^{-1}
d) 98 m s^{-1}
e) 196 m s^{-1}

. . . .

Q15: A steady force of 1200 N is required to slide a crate across the floor at a constant velocity of 2.5 m s^{-1}. At what rate is work being done in moving the crate?

a) 2.1×10^{-3} W
b) 450 W
c) 1200 W
d) 3000 W
e) 3750 W

. . . .

Q16: A projectile of mass 8.0 kg is launched with velocity 18 m s^{-1}, at an angle of 30° to the ground. What is the kinetic energy of the object when it is at its maximum height?

a) 0 J
b) 320 J
c) 970 J
d) 1300 J
e) 7800 J

2.8 Summary

Summary

You should now be able to:

- to state and apply Newton's laws of motion.
- to state that friction is a force and that it always opposes the motion of an object.
- to be able to draw a velocity time graph for an object when friction is taken into account.
- to describe the motion of a rocket as it burns fuel;
- use free body diagrams to analyse the forces acting on an object;
- carry out calculations involving work done, conservation of energy, potential energy, kinetic energy and power.

2.9 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

Links

- This page gives a good account of force diagrams and balanced forces: **http://www.bbc.co.uk/schools/ks3bitesize/science/energy_electricity_forces/forces/revise5.shtml**
- NASA on Newton's laws. This site is worth further exploration if you have time: **http://www.grc.nasa.gov/WWW/K-12/airplane/newton.html**
- Another BBC video clip which demonstrates the friction between a car tyre and the road: **http://www.bbc.co.uk/learningzone/clips/friction-between-a-car-tyre-and-the-road/2179.html**
- This part of the Physics Classroom website explores resolution of forces: **http://www.physicsclassroom.com/class/vectors/u3l3b.cfm**
- This gives a good step by step guide to constructing free body diagrams: **http://www.wisc-online.com/Objects/ViewObject.aspx?ID=tp1502**
- NASA page on the forces on a rocket: **http://exploration.grc.nasa.gov/education/rocket/rktfor.html**
- If you can ignore the adverts, this site will provide detailed information on force, power, torque and energy. Try the link to rollercoasters: **http://auto.howstuffworks.com/auto-parts/towing/towing-capacity/information/fpte9.htm**

- BBC on gravitational potential energy. Questions are provided with answers to reinforce your learning:
  http://www.bbc.co.uk/schools/gcsebitesize/science/add_gateway /forces/themeridesrev1.shtm
- These are excellent interactive simulations which allow the transfer of energy to be investigated.
  http://phet.colorado.edu/en/simulation/energy-skate-park-basics
  http://phet.colorado.edu/en/simulation/energy-skate-park
- This website demonstrates the relationship between the gradient of a velocity time graph and the acceleration:
  http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics /ConstantAccel/ConstantAccel.html

2.10 Assessment

End of topic 2 test Go online

The following test contains questions covering the work from this topic.

The following data should be used when required: ***Acceleration due to gravity g = 9.8 m s^{-2}***

Q17: A 3.8 kg mass is attached to the end of a spring, which is compressed and then released. If the initial acceleration of the mass is 1.3 m s $^{-2}$, calculate the force (in N) in the spring when the mass is released.

. . . .

Q18: A force of 80 N applied to an object causes it to accelerate at 2.0 m s $^{-2}$. Calculate the force in N which is needed to make the object accelerate at 5.4 m s $^{-2}$.

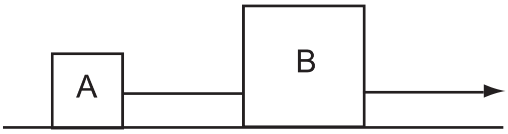
. . . .

Q19: A cyclist exerts an unbalanced force of 160 N as she travels along a level road. The mass of the cyclist and her bicycle is 64 kg.

Starting from rest, calculate how far (in m) the cyclist travels in the first 5.0 s of her journey.

. . . .

Q20: Two boxes on a smooth (frictionless) horizontal surface are joined together by a short length of rope, as shown in the diagram below.



Box A has mass 2.3 kg and box B has mass 6.9 kg. Box B is being pulled by a horizontal force of 14 N in the direction shown.

1. Calculate the acceleration of the blocks, in m s^{-2} .
2. Calculate the tension in the rope connecting A and B , in N.

. . . .

Q21: A box of mass 80 kg is being pulled upwards by a vertical rope.

1. Calculate the tension (in N) in the rope if the box is moving upwards with a constant speed of 6.1 m s^{-1}.
2. Calculate the tension (in N) in the rope if the box is moving upwards with a constant acceleration of 2.0 m s^{-2}.

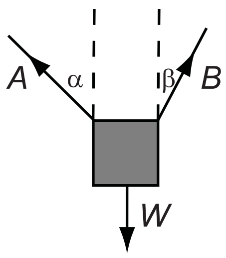
. . . .

Q22: A skier, mass 80 kg, is skiing down a slope that is inclined at 38° to the horizontal. The frictional force acting on the skier is 160 N.

Calculate the acceleration in m s^{-2} of the skier down the slope.

. . . .

Q23: A box of weight W is suspended by two ropes. A free body diagram of the box is given.



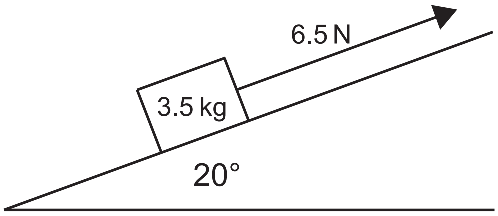
The force A acts at angle α = 45° and the force B acts at angle β = 30° . The weight W of the box is 132 N. The box is stationary. By considering the horizontal and vertical components, calculate:

1. the force A , in N.
2. the force B , in N.

(Hint: you will need to use simultaneous equations to calculate the forces.)

. . . .

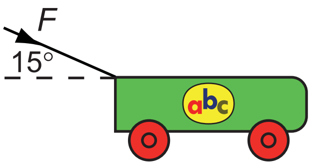
Q24: A 3.5 kg mass is initially at rest on a surface which is inclined at 20° to the horizontal. As the mass slides down the slope, a constant frictional force of 6.5 N acts on it.



Calculate the speed in m s $^{-1}$ of the mass after it has travelled 10 m down the slope.

. . . .

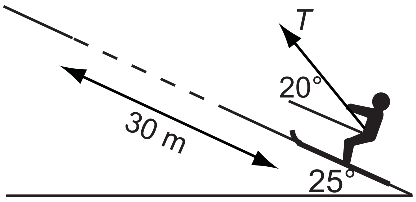
Q25: A child pushes a toy cart with a force F = 6.9 N. The force is applied at an angle of 15°, as shown in the diagram. The mass of the cart is 3.9 kg.



1. Calculate the acceleration of the cart in m s $^{-2}$
2. Calculate the kinetic energy (in J) of the cart when it is travelling at 3.1 m s $^{-1}$.

. . . .

Q26: A skier of weight 640 N is pulled up a slope at a constant speed by a tow rope, as shown in the diagram below.



The slope is inclined at 25° to the horizontal. The tow rope makes an angle of 20° with the slope, and the tension *T* in the tow rope is 520 N. Calculate the potential energy in *J* gained by the skier when she has moved 30 m up the slope.

Topic 3

Collisions, explosions and impulse

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Learning objective

By the end of this section you should be able to:

- define momentum;
- state and apply the law of conservation of momentum;
- show an understanding of the difference between elastic and inelastic collisions;
- state that momentum is conserved during an explosion;
- state and use Newton's third law of motion;
- state that the area under a force time graph is the impulse on the object and is equal to the change in momentum of an object;
- carry out calculations involving momentum, force and impulse;
- draw and interpret force time graphs during contact of colliding objects;
- state that impulse is force times time and that impulse is equal to the change in momentum of an object.

So far we have considered the motion of objects, and what happens when a force is applied to an object. In this topic we will combine some ideas that we have already met to investigate what happens when two objects interact. One of the ways in which objects interact is during collisions.

- We will examine what happens to the momentum of objects during both elastic and inelastic collisions and define elastic and inelastic collisions in terms of kinetic energy.
- We are also going to think about explosions. We will examine what happens to momentum during explosions and then we will think about Newton's Third law of Motion.
- Following looking at collisions and explosions, we will take this a stage further when we examine impulse.
- Impulse relates the change in momentum of an object to the forces acting.

3.1 Momentum

To analyse a collision, we need to introduce a new physical quantity called **momentum**, which depends on the mass and velocity of the object. We will see that using momentum allows us to predict the velocities of objects after they have been involved in collisions or explosions.

Throughout this topic we are going to be concerned with the momentum of different objects. We will be dealing with objects moving in straight lines. The linear momentum p of an object of mass m moving with velocity v is given by Equation 3.1

Momentum is a vector quantity, having both magnitude and direction. The units of p are kg m s^{-1}.

$$p = mv \tag{3.1}$$

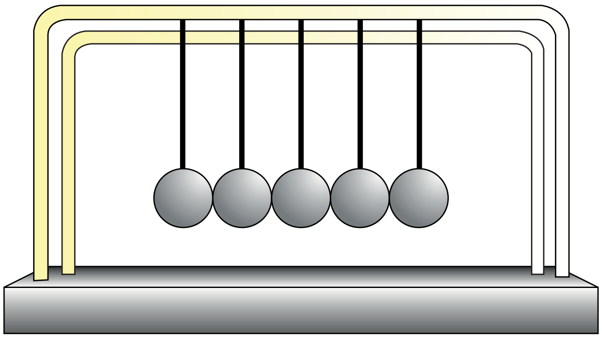
3.2 Types of collisions

To get yourself thinking about collisions and momentum, try the following activity.

Newton's cradle Go online

At this stage there is an online activity. If you do not have access to the internet read the following explanation and make sure that you understand it.

You may already be familiar with Newton's cradle.



When we pull back one ball and release it a collision takes place and the ball at the other end of the cradle is pushed out and up. It falls back down and the process repeats itself over and again.

If we pull back two balls instead of one the last two balls at the far end are pushed out. These fall back down and the process repeats itself with two balls always being pushed out of the ends of the cradle.

If we use three balls the process repeats with three balls always being pushed out. The same thing happens for four balls.

How can we use momentum to explain this?

When we pull one ball back we are giving the system one unit of momentum so after the collision it must also have one unit of momentum. This means that one ball is pushed out of the cradle.

If we pull back two balls we are giving the system two units of momentum so after the collision there must be two units of momentum so two balls must move.

The same is true when we use three and four balls.

The cradle is obeying the law of conservation of linear momentum: in the absence of external forces the total momentum before a collision is equal to the total momentum after the collision.

In any collision, total momentum is conserved.

When two objects collide, the law of **conservation of momentum** states that the total momentum is conserved, so long as no external force acts on the two objects. That is to say, the vector sum of the momentum of the two objects before the collision is equal to the vector sum of their momentum after the collision. If you look back at the *Newton's cradle* interactivity you will see that momentum is being conserved in every collision - the same number of balls move with the same velocity before and after a collision.

Throughout this topic, we will only be concerned with objects moving in one dimension.

A more thorough treatment would show us that momentum is conserved in any direction.

To clarify matters, the labelling convention used throughout this Topic is as follows: for a collision between two objects of masses m_1 and m_2, their velocities before the collision are labelled u_1 and u_2 respectively. After the collision, m_1 has velocity v_1 and m_2 has velocity v_2. The law of conservation of momentum can then be summarised in Equation 3.2.

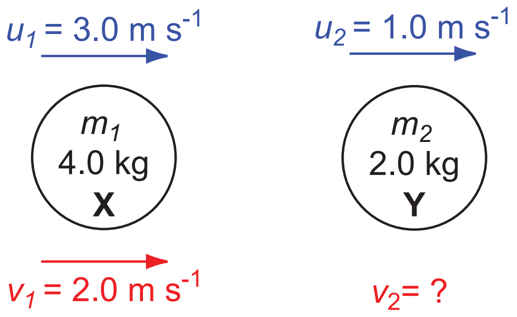
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{3.2}$$

Remember that momentum and velocity are vector quantities. All velocities are measured in the same dimension, and therefore may be assigned a positive or negative value.

A sketch diagram is often useful. Consider two situations involving collisions between two spheres *X* and *Y*, with m_1 = 4.0 kg and m_2 = 2.0 kg. Suppose *X* is moving at 3.0 m s^{-1} and *Y* is moving at 1.0 m s^{-1}. Both spheres are moving in the same direction.

After they collide, *X* continues in the same direction with velocity 2.0 m s^{-1}. What is the new velocity of *Y*? A sketch diagram of this situation is shown in Figure 3.1.

Figure 3.1: Sketch diagram of the two colliding spheres



Using the diagram, we can straight away write down and solve the conservation of momentum equation.

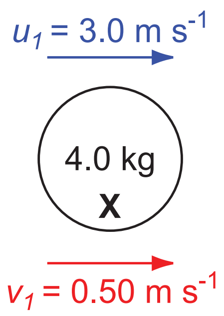
$$\begin{aligned} m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\ \therefore (4.0 \times 3.0) + (2.0 \times 1.0) &= (4.0 \times 2.0) + (2.0 \times v_2) \\ \therefore 14 &= 8.0 + 2v_2 \\ \therefore v_2 &= 3.0 \text{ m s}^{-1} \end{aligned}$$

So after the collision, *Y* has velocity 3.0 m s^{-1} in its original direction.

Suppose instead the two spheres were moving *towards* each other, with *X* travelling at 3.0 m s^{-1} and *Y* moving at 1.0 m s^{-1}. If *X* has a velocity after the collision of 0.50 m s^{-1} in the same direction as it was originally moving, what is the new velocity of *Y*?

Again, a sketch diagram like the one shown in Figure 3.2 is useful.

Figure 3.2: Sketch diagram for two spheres moving towards each other



Note that this time sphere *Y* is initially moving to the left so its velocity is a negative value. Using the labelling shown in Figure 3.2 should ensure you do not make a mistake with the signs of the velocities in the conservation of momentum equation:

$$\begin{aligned} m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\ \therefore (4.0 \times 3.0) + (2.0 \times -1.0) &= (4.0 \times 0.50) + (2.0 \times v_2) \\ \therefore 10 &= 2.0 + 2v_2 \\ \therefore v_2 &= 4.0 \text{ m s}^{-1} \end{aligned}$$

After this collision, the velocity v_2 is positive 4.0 m s^{-1}. Since the velocity is positive it tells us that sphere *Y* is moving to the right at 4.0 m s^{-1}.

Collision between two spheres

A sphere *A* of mass 2.0 kg rolls into a stationary sphere *B* of mass 3.0 kg. The velocity of *A* before the collision is 5.0 m s^{-1} and its velocity after the collision is 0.50 m s^{-1}.

Sketch a diagram and hence calculate the velocity of *B* after the collision.

A sketch diagram showing the correct velocity vectors is useful in solving problems involving collisions.

3.2.1 Inelastic collisions

As well as studying the momentum of objects colliding with one another, we can also consider their kinetic energy. If you look at the first example in this topic, the kinetic energy of sphere *X* before the collision is

$$\begin{aligned} E_{k1} &= \frac{1}{2}m_1{u_1}^2 \\ \therefore E_{k1} &= \frac{1}{2} \times 4.0 \times 3.0^2 \\ \therefore E_{k1} &= 18 \text{ J} \end{aligned}$$

Using a $E_{k2} = \frac{1}{2}mu^2$ calculation, sphere *Y* has kinetic energy 1.0 J before the collision. (Make sure you can perform this calculation.) So the total kinetic energy before the collision is 18 + 1.0 = 19 J.

After the collision, the kinetic energy of sphere *X* is $E_{k1} = \frac{1}{2}m{v_1}^2$ and kinetic energy of sphere *Y* is $E_{k2} = \frac{1}{2}m{v_2}^2$. If you carry out the calculations, you should find the new kinetic energy of *X* is 8.0 J and the new kinetic energy of *Y* is 9.0 J. The total kinetic energy after the collision is 8.0 + 9.0 = 17 J. So the *total* energy has been reduced by 2.0 J, from 19 J to 17 J.

A collision in which kinetic energy is not conserved is called an **inelastic collision**.

It is crucial to note that the term inelastic collision refers only to a change in kinetic energy. The total momentum before the collision will however equal the total momentum after the collision; total momentum will have been conserved.

If a collision is inelastic then all that can be stated is that the total kinetic energy has not been conserved. Inelastic has nothing to do with objects joining or not joining together.

In a collision where the two objects stick together after colliding they will have the same velocity after the collision. The conservation of momentum equation becomes:

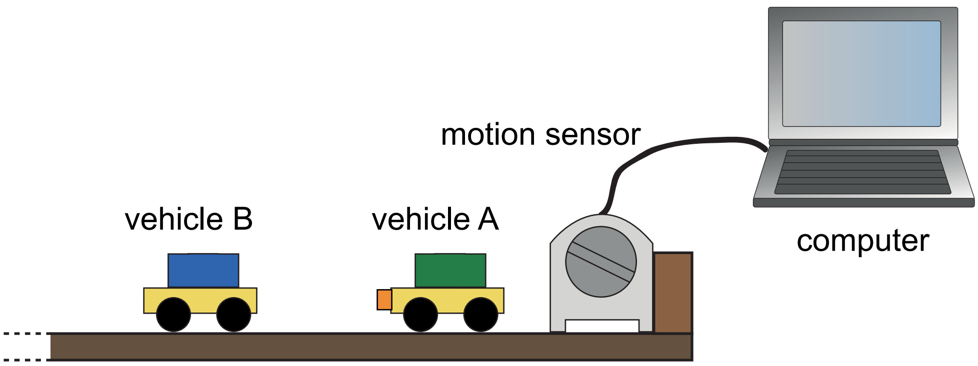
$$m_1u_1 + m_2u_2 = (m_1+m_2)v \tag{3.3}$$

Any collision in which momentum is conserved and kinetic energy is not conserved is called an inelastic collision.

Inelastic collisions Go online

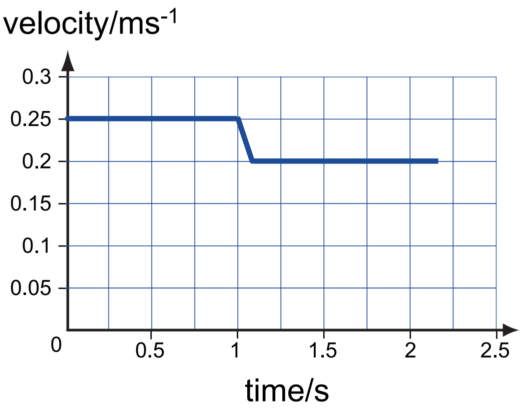
At this stage there is an online activity on inelastic collisions. If however you do not have access to the internet you may try the question which follows.

Q1: The apparatus shown is set up to investigate collisions between two vehicles on a track.



The mass of vehicle A is 0·22 kg and the mass of vehicle B is 0·16 kg. The effects of friction are negligible.

During one experiment the vehicles collide and stick together. The computer connected to the motion sensor displays the velocity-time graph for vehicle A.

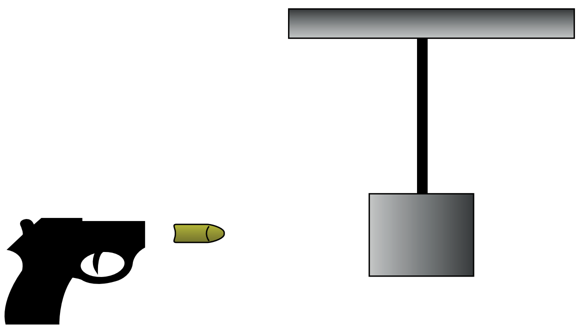


1. Calculate the velocity of vehicle B before the collision.
2. Calculate the loss in kinetic energy during the collision.
3. What has happened to this kinetic energy?
4. The same apparatus is used to carry out a second experiment.
   In this experiment, vehicle B is stationary before the collision. Vehicle A has the same velocity before the collision as in the first experiment. After the collision, the two vehicles stick together.
   Is their combined velocity less than, equal to, or greater than that in the first collision?

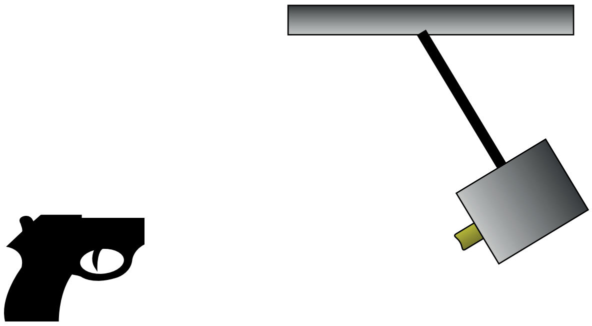
Ballistic pendulum Go online

At this stage there is an online activity. If however you do not have access to the internet you may read the explanation and try the questions which follow.

A shotgun pellet is fired into a block of wood, which is suspended by a thread from the ceiling. By measuring the height to which the block and embedded bullet swing, can you calculate the velocity of the bullet fired from the gun?



The mass of the bullet is equal to 0.020 kg and the mass of the block to 2.0 kg. The bullet is now fired at the pendulum and the maximum height it rises through is measured



Work through the calculations in the questions which follow to deduce the velocity at which the bullet is fired from the gun.

Q2: The maximum height to which the block with the embedded bullet swings is 0.42 m. Calculate the gain in potential energy of the combined block and bullet.

. . . .

Q3: Using the previous answer, state the kinetic energy of the combined block and bullet just after the bullet embeds itself in the block.

. . . .

Q4: Calculate the velocity of the combined block and bullet just after the bullet embeds itself in the block.

. . . .

Q5: Use conservation of momentum to calculate the velocity of the bullet before it hits the block.

In an inelastic collision, momentum is conserved but kinetic energy is not.

3.2.2 Elastic collisions

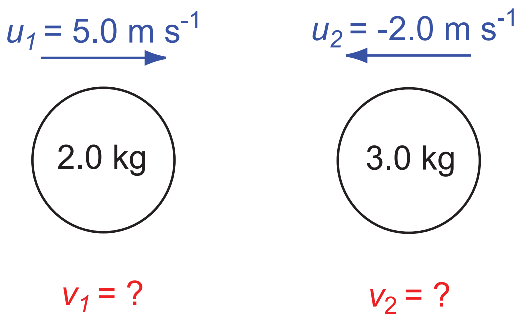
An **elastic collision** between two objects is one in which both momentum and kinetic energy are conserved. Since two quantities are conserved, we can write two "before and after" equations to solve problems involving elastic collisions.

Example

Let us consider two spheres *1* (2.0 kg) and *2* (3.0 kg). Suppose they are moving towards each other, with *1* travelling at 5.0 m s^{-1} and *2* travelling at 2.0 m s^{-1}. Find the velocities of *1* and *2* if they undergo an elastic collision.

Answer:

Figure 3.3: Sketch diagram for elastic collision



The conservation of momentum equation is

$$\begin{aligned} m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\ \therefore (2 \times 5) + (3 \times -2) &= (2 \times v_1) + (3 \times v_2) \\ \therefore 4 &= 2v_1 + 3v_2 \\ \therefore v_1 &= 2 - \tfrac{3}{2}v_2 \end{aligned}$$

The conservation of kinetic energy equation is

$$\begin{aligned} \tfrac{1}{2}m_1{u_1}^2 + \tfrac{1}{2}m_2{u_2}^2 &= \tfrac{1}{2}m_1{v_1}^2 + \tfrac{1}{2}m_2{v_2}^2 \\ \therefore m_1{u_1}^2 + m_2{u_2}^2 &= m_1{v_1}^2 + m_2{v_2}^2 \\ \therefore \left(2 \times 5^2\right) + \left(3 \times -2^2\right) &= \left(2 \times {v_1}^2\right) + \left(3 \times {v_2}^2\right) \\ \therefore 62 &= 2{v_1}^2 + 3{v_2}^2 \end{aligned}$$

We can substitute for v_1 in this equation.

$$62 = 2\left(2 - \tfrac{3}{2}v_2\right)^2 + 3{v_2}^2$$

This expression can be expanded and simplified, leading to a quadratic equation in v_2.

$$5{v_2}^2 - 8v_2 - 36 = 0$$

Make sure you can perform the expansion and simplification to arrive at this stage. The solutions to this quadratic equation are v_2 = -2.0 m s^{-1} and v_2 = 3.6 m s^{-1}. The first solution can be discarded, since this solution corresponds to sphere A initially starting to the right of B. In this case, the spheres move apart and do not collide. The solution v_B = 3.6 m s^{-1} can be substituted back into the conservation of momentum equation to give the final answers: v_1 = -3.4 m s^{-1} and v_2 = 3.6 m s^{-1}.

Elastic collisions Go online

At this stage there is an online activity. If however you do not have access to the internet you may try the question which follows.

Q6: Two identical trolleys each with a mass of 4.0 kg are sitting stationary on a frictionless track. The first trolley is given a velocity of 2.0 ms^{-1} towards the second trolley. The trolleys then collide. The first trolley stops as a result of the collision.

1. What is the velocity of the second trolley after the collision?
2. Show by calculation that the collision is elastic

Quiz: Momentum Go online

Q7: Ball *M* (mass 0.500 kg) moves at 3.00 m s^{-1} into ball *N*, which is stationary and has mass 0.900 kg. If the collision brings *M* to rest, what is the velocity of *N* immediately after the collision?

a) 0.120 m s^{-1}
b) 0.250 m s^{-1}
c) 1.08 m s^{-1}
d) 1.67 m s^{-1}
e) 6.75 m s^{-1}

. . . .

Q8: A 2500 kg train carriage travelling at 10 m s^{-1} runs into the back of a stationary carriage of mass 4000 kg. If the carriages couple together, what is their velocity just after impact?

a) 3.8 m s^{-1}
b) 6.2 m s^{-1}
c) 6.3 m s^{-1}
d) 5.0 m s^{-1}
e) 10 m s^{-1}

. . . .

Q9: In any inelastic collision between two objects,

a) total kinetic energy is conserved but total momentum is not.
b) both objects must coalesce after impact.
c) total momentum and total kinetic energy are both conserved.
d) one object must be stationary before the collision.
e) total momentum is conserved but total kinetic energy is not.

. . . .

Q10: An object is travelling with momentum 100 kg m s^{-1} and kinetic energy 80 J. What is the velocity of the object?

a) 0.89 m s^{-1}
b) 1.3 m s^{-1}
c) 1.6 m s^{-1}
d) 7.9 m s^{-1}
e) 63 m s^{-1}

....

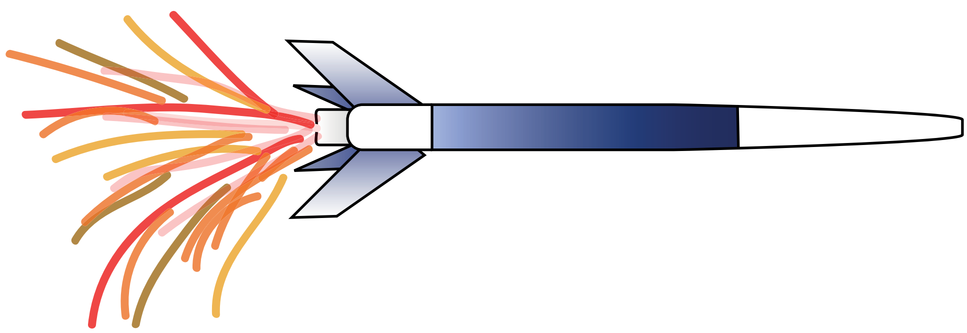
Q11: Two spheres, one of mass 4.0 kg and the other of mass 1.0 kg, collide head-on. Each is moving at 3.0 m s^{-1} before the collision. If the collision is perfectly elastic, what is the speed of the 4.0 kg sphere immediately after the collision?

a) 0.60 m s^{-1}
b) 1.8 m s^{-1}
c) 3.0 m s^{-1}
d) 3.2 m s^{-1}
e) 6.6 m s^{-1}

3.3 Explosions

When a gun is fired, the bullet shoots out of the barrel, and the marksman feels a *kick* or recoil from the gun. This is another example of conservation of momentum. Before the gun is fired, the gun and the loaded bullet are both stationary. Immediately after it is fired, the bullet has momentum in the direction it is travelling. To keep the total momentum at zero, the gun must recoil in the opposite direction.

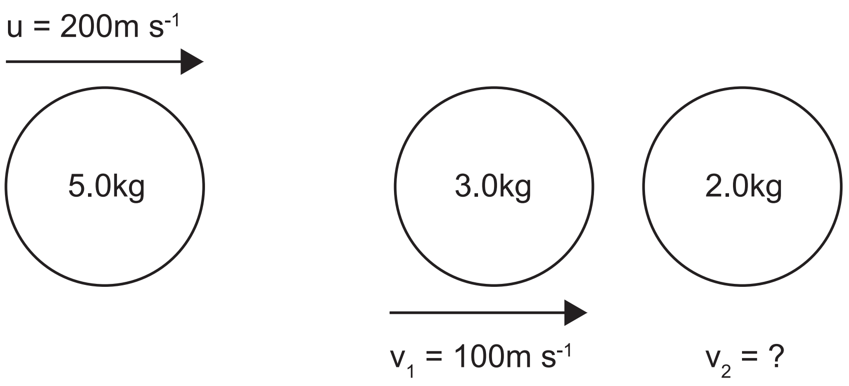
Rockets and jet engines are two more practical examples of the conservation of momentum in explosions. Let us look first at the rocket. The fuel and oxidant are both stored on the rocket, and a portion of these are used up every unit of time. They are expelled at high speed from the rear of the rocket as exhaust gases, causing an increase in the forward momentum of the rocket.



A jet engine takes in cool air at its front end. This air is compressed, and used in the engine to support the burning of the engine fuel. Again, the air and the exhaust gases are expelled from the rear of the engine, resulting in an increase in the forward momentum of the aeroplane.

Example A shell of mass 5.0 kg is travelling horizontally with a speed of 200 m s^{-1}. It explodes into two parts. One part of mass 3.0 kg continues in the original direction with a speed of 100 m s^{-1}. Calculate the velocity of the 2.0 kg part.

Using a similar method of sketch diagram as before, this explosion can be shown as:



Its speed is $m\,u = m_1v_1 + m_2v_2$

The shell is in one piece before the explosion so

$m\,u = m_1v_1 + m_2v_2$

$5 \times 200 = 3 \times 100 + 2v_2$

$2v_2 = 700$

$v_2 = 350$ m s^{-1}

The velocity is positive so it is in the same direction as the shell was travelling originally.

3.4 Newton's third law of motion

When you hit a ball with a bat you exert a force on the ball. But the ball exerts a force back on the bat. This force is equal in size to the original force but in the opposite direction.

Newton's third law of motion states that *for every action there is an equal but opposite reaction*.

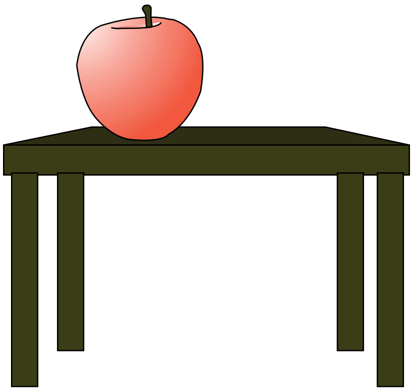
When the rocket in the example above pushes the exhaust gases out of the rear of the rocket it exerts a force on them. As a result on Newton's third law of motion the gases exert a force on the rocket that is equal in size but opposite in direction to the force of the rocket on the gases. This causes the rocket to move forwards.

Forces always occur in pairs. At the moment if you are sitting on a chair you are exerting a force on the chair. The chair exerts an equal but opposite force back on you.

Examples

1. Apple

An apple sits on a desk.



The apple exerts a force on the desk. Name the reaction force.

The force of the desk on the apple.

. . . .

2. Woman

A woman walks along a road.

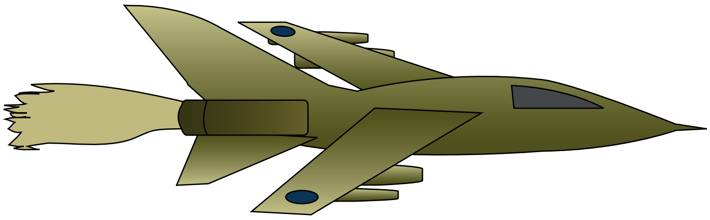
The woman pushes backwards against the road. Why does she move forward?

The road exerts an equal but opposite force on the woman so she moves forward.

. . . .

3. Jet engine

A jet engine is used to power a fighter aircraft.

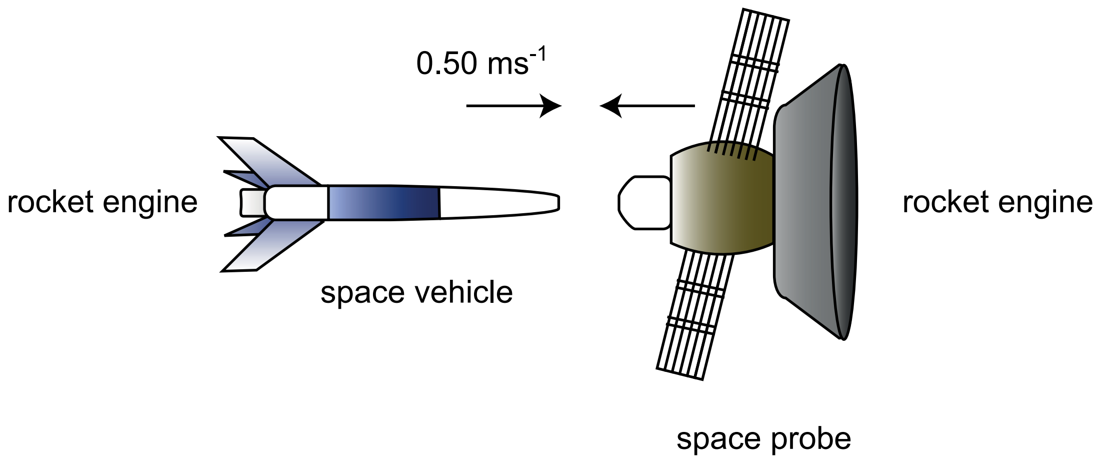


Explain how the jet engine powers the aircraft.

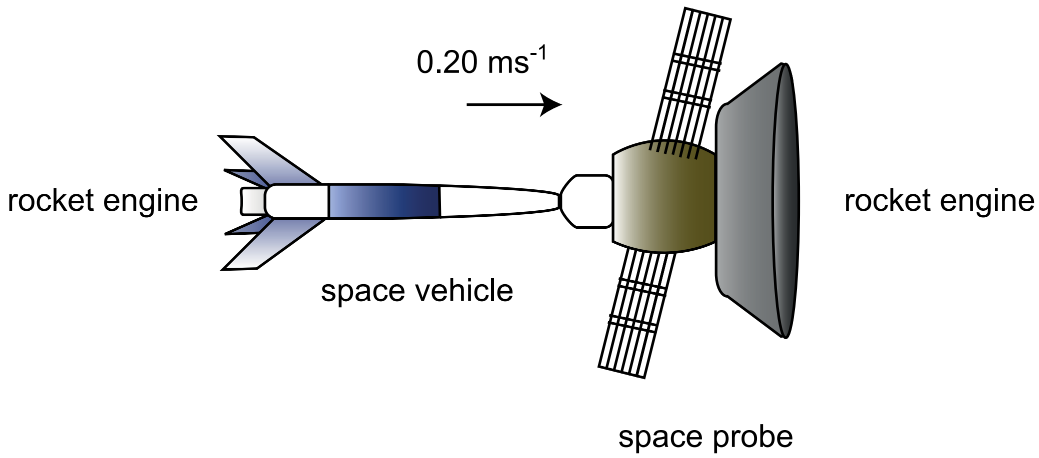
The jet engine pushes gases backwards so the gases exert an equal but opposite force on the aircraft pushing it forwards.

Rocket

A space vehicle of mass of 2500 kg is moving with a constant speed of 0.50 m s^{-1} in the direction shown. It is about to dock with a space probe of mass 1500 kg which is moving with a constant speed in the opposite direction.



After docking the space vehicle and probe move off together at 0.20 m s^{-1} in the original direction in which the space vehicle was moving.



1. Calculate the speed of the space probe before it docked with the space vehicle.
2. The space vehicle has a rocket engine which produces a constant thrust of 1000 N. The space probe has a rocket engine which produces a constant thrust of 500 N. The space vehicle and probe are now brought to rest from their combined speed of 0.20 m s^{-1}
    1. Which rocket engine was switched on to bring the vehicle and probe to rest?
    2. Calculate the change in momentum of the space vehicle and probe.
    3. What is the change in momentum of the exhaust gases from rocket engine?

Quiz: Explosions Go online

Q12: Two blocks A (1.5 kg) and B (2.5 kg) are at rest on a smooth horizontal surface. A light spring is compressed between A and B. When the blocks are released, A moves to the left at 4.0 m s^{-1}. What is the speed of B to the right?

a) 0.42 m s^{-1}
b) 1.1 m s^{-1}
c) 2.4 m s^{-1}
d) 4.0 m s^{-1}
e) 6.7 m s^{-1}

. . . .

Q13: A gun fires a bullet of mass 0.020 kg at 480 m s^{-1}. If the mass of the gun is 0.80 kg, what is the recoil velocity of the gun?

a) 0.083 m s^{-1}
b) 0.24 m s^{-1}
c) 4.8 m s^{-1}
d) 7.7 m s^{-1}
e) 12 m s^{-1}

. . . .

Q14: A nucleus of mass *m* splits into two particles, one of mass 0.25*m*, the other of mass 0.75*m*. If the less massive particle has speed *v*, what is the speed of the other particle, travelling in the opposite direction?

a)
$$\frac{v}{3}$$
b)
$$\frac{2v}{3}$$
c)
$$\frac{3v}{4}$$
d)
$$\frac{4v}{3}$$
e)
$$\frac{3v}{2}$$

. . . .

Q15: At the bowling alley, a woman of mass 45 kg is using a 5.0 kg ball. She runs up at 2.0 m s^{-1} and bowls the ball at 10 m s^{-1}. What is her velocity immediately after releasing the ball?

a) 0.67 m s^{-1}
b) 0.89 m s^{-1}
c) 0.91 m s^{-1}
d) 1.1 m s^{-1}
e) 3.3 m s^{-1}

. . . .

Q16: When a cannon fires a 10 kg cannonball at 30 m s^{-1}, its recoil velocity is 6.0 m s^{-1}. What is the recoil velocity when a 15 kg cannonball is fired at 24 m s^{-1}?

a) 1.2 m s^{-1}
b) 6.0 m s^{-1}
c) 7.2 m s^{-1}
d) 31 m s^{-1}
e) 36 m s^{-1}

3.5 Impulse

Can we link the work we have done on momentum in this topic to Newton's laws of motion?

Clearly if two objects collide, at the moment of impact each object exerts a force on the other, and Newton's third law tells us that these forces are equal in magnitude and opposite in direction.

Let us look at the force acting on one of the objects in the collision. Newton's second law tells us that the force F acting on it causes an acceleration, given by the equation
F = ma

We have previously defined acceleration by the equation $a = \frac{v-u}{t}$

$$\begin{aligned} F &= ma \\ \therefore F &= \frac{m(v-u)}{t} \\ \therefore F &= \frac{mv-mu}{t} \end{aligned} \tag{3.4}$$

- *mv - mu* is the change in momentum,
- *t* is the time for the change in momentum to occur.

So force is the rate of change of momentum. Rearranging Equation 3.4,

$$F \times t = mv - mu$$

The quantity $F \times t$ is called the **impulse**. You should be able to state that impulse is the change of momentum. Impulse has units of N s or kg m s^{-1}.

Example : Parking space

A car driver trying to reverse into a parking space crashes into a wall. The car, mass 1700 kg, is travelling at 2.50 m s^{-1} as it hits the wall. What is the impulse acting on the car?

Answer:

Impulse is defined as the change in momentum:

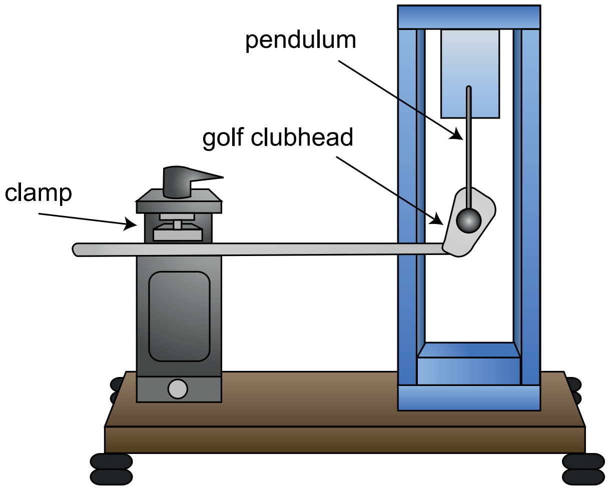
$$\text{impulse} = mv - mu$$
$$\therefore \text{impulse} = (1700 \times 0) - (1700 \times 2.5)$$
$$\therefore \text{impulse} = -4250 \text{ kg m s}^{-1}$$

The impulse on the car is 4250 kg ms^{-1}.

In this case the impulse is negative as the final momentum of the car is less than its momentum before the collision.

This example highlights one of the practical applications of momentum and impulse, which is in the design of car bumpers and *crumple zones*. With safety in mind, these parts of a car are designed to give way in a collision. If a car is driven into a wall, the car comes to a halt pretty quickly, with the occupants of the car continuing to move at the same speed. The crumple zone means that the collision is not one between two inflexible objects - because the car is designed to crumple, the collision is spread out in time, resulting in a smaller force acting on the people in the car.

Example Golf clubs are tested to ensure they meet certain standards.



In one test, the machine uses a club to hit a stationary golf ball. The mass of the ball is 4.5×10^{-2} kg. The ball leaves the club with a speed of 50.0 m s^{-1}. The time of contact between the club and ball is 450 μs.

1. Calculate the average force exerted on the ball by the club.

2. The test is repeated using a different club and an identical ball. The machine applies the same average force on the ball but with a longer contact time. What effect, if any, does this have on the speed of the ball as it leaves the club?

Justify your answer.

1. F = m (v - u)/t

   F = 4.5 $\times$ 10^{-2}(50 - 0)/ 450 $\times$ 10^{-6}

   F = 5000 N

   The force on the ball is 5000 N.
2. The speed has increased because the impulse (F $\times$ t) on the ball is greater.

Impulse is the change in momentum of an object.

Test your strength contest Go online

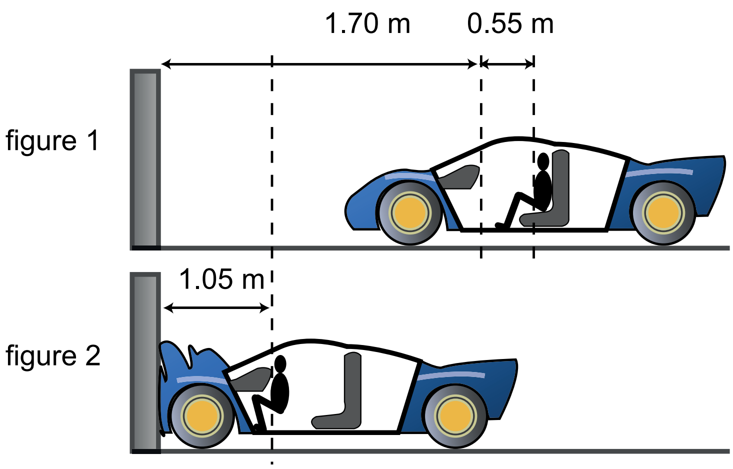
At this stage there is an online activity.

If you do not have access to the web you can try the following question. A hint and an answer are provided for you to check your solution against.

The metal block has mass 1.0 kg, and has to rise to a height of 5.0 m to reach the top of the pole. The hammer has mass 1.5 kg. If the impulse applied in bringing the hammer to rest is equal to the impulse applied to the block, calculate the minimum downward velocity of the hammer which is necessary to ensure the block reaches the top of the pole.

Car safety

In a test a dummy passenger is placed, without a seat belt, in a car 0.55 m from the dashboard. The car is made to crash into a rigid wall at a speed of 12 m s^{-1}. The front of the car crumples progressively and shortens by 0.65 m before the car comes to rest. The passenger compartment is not damaged.



During the crash the dummy continues forward freely.

Q17: At what speed does the dummy hit the dashboard, which has already come to rest?

. . . .

Q18: How far does the dummy travel, relative to the ground, between the instant when the front of the car first makes contact with the wall (figure 1) and the instant when the dummy first makes contact with the dashboard (figure 2)?

. . . .

Q19: What time elapses between these two instants?

. . . .

Q20: The dummy has a mass of 80 kg. Calculate the change in momentum of the dummy.

. . . .

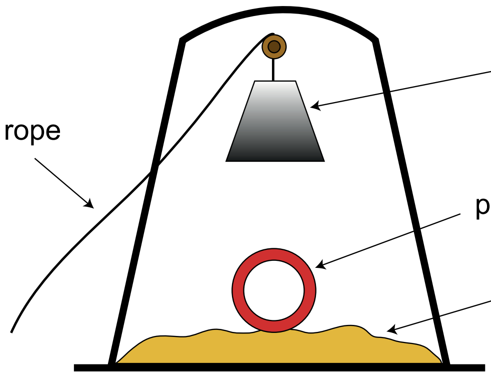
Q21: The collision between the dummy and the dashboard takes 0.05 s. Calculate the average force on the dummy.

. . . .

Q22: Modern cars are fitted with air bags. Explain how an air bag would reduce the size of the force acting on the dummy.

Pile driver

A pile driver is used to test concrete pipes as shown.



When the rope is released, the 15 kg mass is dropped and falls freely through a distance of 2.0 m on to the pipe. When the mass hits the pipe it comes to rest. In one test, the mass is dropped on to an uncovered pipe.

Q23: Calculate the speed of the mass just before it hits the pipe.

. . . .

Q24: Calculate the change in momentum of the 15 kg mass when it hits the pipe.

. . . .

Q25: When the mass hits the pipe it is brought to rest in a time of 0.02 s. Calculate the size and direction of the average force on the **pipe**.

. . . .

Q26: The pipe is covered with a thick layer of soft material and the experiment is repeated. Describe and explain the effect this layer has on the average force on the pipe.

3.6 Force time graphs

You need internet access for the following.

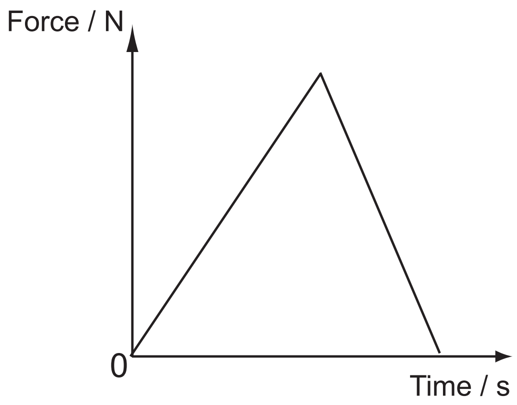
This link **http://www.youtube.com/watch?v=2Y57pw_iWlk** brings you to a video clip shows a golf ball being compressed as it is hit by the club. The force on the ball (and on the club) increases as the ball becomes more compressed. As the ball begins to return to its normal shape the force decreases.

This is change in force is represented in the force time graph shown above. The maximum force is when the ball is most compressed.

When an unbalanced force acts on an object it accelerates. This means that it undergoes a change in momentum.

In the equation Impulse = force $\times$ time the force is actually the average force acting for the time.

If we draw a graph of the force that a bat exerts on a ball we typically obtain a graph with the following shape.

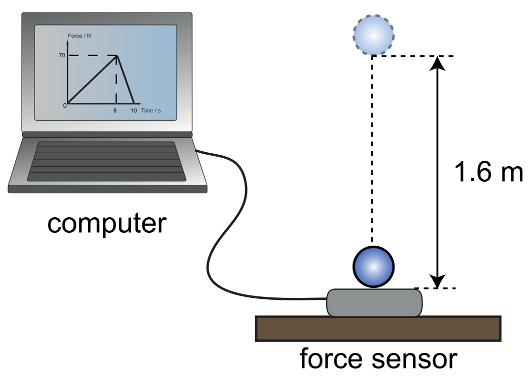


The area under this graph is the impulse. The impulse is also equal to the change in momentum.

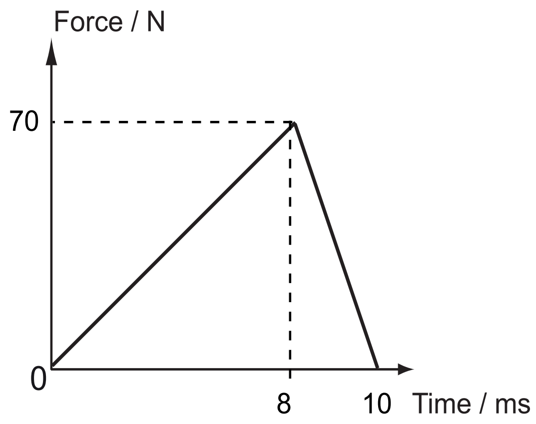
The above graph can be used therefore to find the change in momentum of ball.

Because total momentum is always conserved then the change in momentum in the bat will have the same size as the change in momentum on the ball but in the opposite direction.

Example A force sensor is used to investigate the impact of a ball as it bounces on a flat horizontal surface. The ball has a mass of 0.050 kg and is dropped vertically, from rest, through a height of 1.6 m as shown.



A) 1. The graph shows how the force on the ball varies with time during the impact.



Show that the magnitude of the impulse on the ball is 0.35 Ns.

2. What is the magnitude and direction in the change in momentum of the ball?
3. The ball is travelling at 5.6 m s^{-1} just before it hits the motion sensor. Calculate the speed of the ball just as it leaves the force sensor.

B) Another ball of identical size and mass, but made of a harder material, is dropped from rest and from the same height onto the same force sensor. This harder ball rebounds to the same height as the softer ball. Sketch the force time graph above and, on the same axes, sketch another graph to show how the force on the harder ball varies with time.

Numerical values are not required but you must label the graphs clearly.

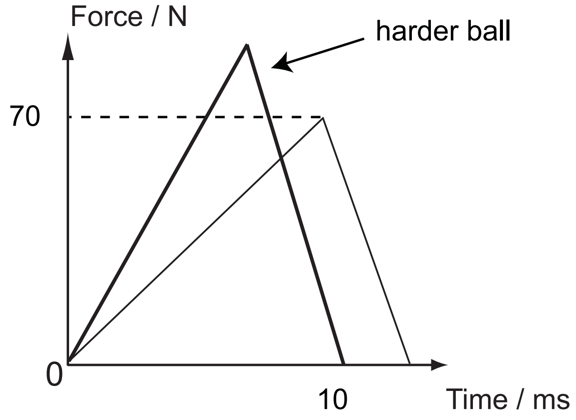
A) 1. Impulse = area under F-t graph = 1/2 base $\times$ height = 1/2 $\times$ 0.010 = $^1/_2 \times$ 0.010 $\times$ 70 = 0.35 Ns

- Usual layout of calculation could be:
- insert return after 1/2 x base x height
- align = 1/2 x 0.010 x 70 with line above
- return after 70
- align = answer with above

The impulse on the ball is 0.35 N s

2. Change in momentum = 0.35 kg m s^{-1} upwards
3. Impulse = mv - mu

0.35 = 0.050x(v - (-5.6))
v = 1.4 m s^{-1}
The speed of the ball as it leaves the force sensor is 1.4 m s^{-1}.



B)

Quiz: Impulse Go online

Q27: The velocity of a 640 kg racehorse increases from 12 m s^{-1} to 17 m s^{-1}. What is the impulse on the racehorse?

a) 204 kg m s^{-1}
b) 3200 kg m s^{-1}
c) 7680 kg m s^{-1}
d) 10880 kg m s^{-1}
e) 18560 kg m s^{-1}

. . . .

Q28: Newton's second law can be expressed as

a) force = impulse.
b) impulse = rate of change of momentum.
c) force = rate of change of momentum.
d) impulse = mass $\times$ acceleration.
e) force = impulse $\times$ time.

. . . .

Q29: A horizontal force of 30 N acts on a 0.40 kg mass at rest on a smooth table top. What is the momentum of the mass after 2.0 s?

a) 6.0 kg m s^{-1}
b) 12 kg m s^{-1}
c) 24 kg m s^{-1}
d) 30 kg m s^{-1}
e) 60 kg m s^{-1}

. . . .

Q30: Impulse has the same unit as

a) momentum.
b) force.
c) velocity.
d) acceleration.
e) energy.

. . . .

Q31: The brakes of a car are applied for 4.0 s, reducing the velocity of the car from 20 m s^{-1} to 8.0 m s^{-1}. If the mass of the car is 1600 kg, what is the magnitude of the force exerted by the brakes?

a) 33 N
b) 530 N
c) 3200 N
d) 4800 N
e) 8000 N

3.7 Summary

Summary

You should now be able to:

- define momentum;
- state and apply the law of conservation of momentum;
- show an understanding of the difference between elastic and inelastic collisions;
- state that momentum is conserved during an explosion;
- state Newton's third law of motion;
- carry out calculations involving momentum, force and impulse;
- draw and interpret force time graphs during contact of colliding objects;
- state that the area under a force time is the impulse on the object and is equal to the change in momentum of an object;
- to draw and interpret force-time graphs during contact of colliding objects.

3.8 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

Links

- Good for back up material and revision purposes:
http://www.sparknotes.com/testprep/books/sat2/physics/chapter9section3.rhtml
- A concise explanation of momentum and impulse. This is a good site to explore for other content related to this course:
http://www.splung.com/content/sid/2/page/momentum
- A good summary and great diagrams on this site:
http://www.s-cool.co.uk/a-level/physics/momentum-and-impulse/revise-it/principle-of-the-conservation-of-momentum
- Some excellent video clips showing conservation of momentum and the effect of increasing the time of contact:
http://physicsnet.co.uk/a-level-physics-as-a2/further-mechanics/momentum-concepts/
- Velocity and kinetic energy values. Ignore advanced tab as it is beyond Higher level:
https://phet.colorado.edu/sims/collision-lab/collision-lab_en.html

- Shows collisions when two cars do not join. Use figures to prove that total momentum is conserved:
  https://sites.google.com/site/physicsflash/home/collision

3.9 Assessment

End of topic 3 test Go online

The following test contains questions covering the work from this topic.

The following data should be used when required:
Acceleration due to gravity g = 9.8 m s^{-2}

Q32: A bullet of mass 10 grams is fired horizontally into a wooden block of mass 0.91kg, which is at rest on a smooth friction free table. The bullet embeds itself in the block, which slides along the table at 1.5 m s^{-1} .

Calculate the velocity in m s^{-1} of the bullet before it strikes the block.

. . . .

Q33: Block A (mass 0.50 kg) is moving at 1.2 m s^{-1} on a frictionless surface when it collides head on with the stationary block B (mass 0.40 kg). After the collision, A continues moving in the same direction at 0.20 m s^{-1} .

Calculate how much kinetic energy (in J) is lost in the collision.

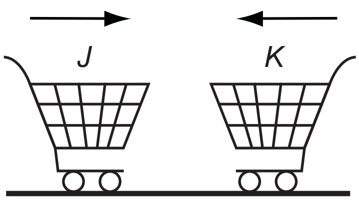
. . . .

Q34: Two identical solid spheres X and Y are rolling towards each other. X has velocity +5.0 m s^{-1} and Y has velocity -3.0 m s^{-1} . Each sphere has mass 4 kg. The spheres collide elastically.

1. State the velocity in m s^{-1} of X after the collision.
2. State the velocity in m s^{-1} of Y after the collision.

. . . .

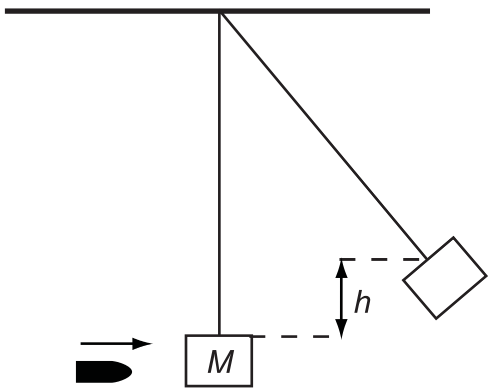
Q35: Two trolleys are rolling towards each other, as shown in the diagram. Trolley J has momentum 23 kg m s^{-1}, whilst trolley K has momentum 16 kg m s^{-1} in the opposite direction.



Both trolleys change their direction after the collision. If the momentum of J after the collision is 4.0 kg m s^{-1}, calculate the magnitude of the new momentum in kg m s^{-1} of K .

. . . .

Q36: A bullet is fired into a wooden block which is suspended from a long thread, as shown in the diagram.

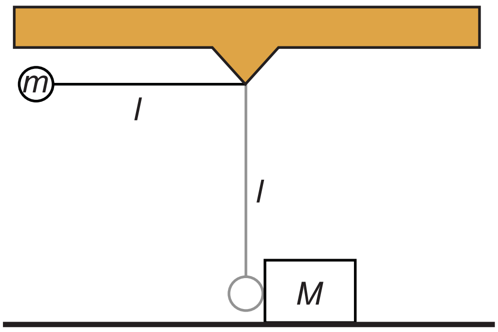


The bullet (mass 15 grammes) is travelling at 310 m s $^{-1}$ when it embeds itself in the block. The block has mass M = 2.4 kg, and rises to a maximum vertical height *h* m.

Calculate the value of *h* , in m.

. . . .

Q37: A ball of mass m = 1.2 kg is attached to a rope of length *l* = 40 cm. The other end of the rope is attached to a rigid support, as shown in the diagram.

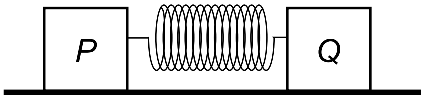


The ball is released from rest when the rope is horizontal. The rope remains taut as the ball swings down, colliding with a wooden block of mass M = 2.4 kg which is stationary on a smooth horizontal surface. The ball strikes the block when the rope is vertical, and comes to rest in the collision.

Calculate the velocity in m s $^{-1}$ of the block immediately after it is struck by the ball.

. . . .

Q38: The diagram shows two masses P (0.50 kg) and Q (0.80 kg) at rest on a smooth horizontal surface. A light spring is compressed between the masses. When they are released, P moves to the left with speed 9.2 m s $^{-1}$.



Calculate the speed with which Q moves to the right, in m s $^{-1}$.

. . . .

Q39: A shell of mass 7.0 kg is travelling horizontally at 190 m s $^{-1}$ when it breaks up into two parts. One part, mass 4.0 kg, continues moving at 140 m s $^{-1}$ without changing direction.

Calculate the velocity in m s $^{-1}$ of the second piece of the shell.

. . . .

Q40: A golfer strikes a stationary golf ball of mass 0.045 kg, giving the ball a velocity of 29 m s $^{-1}$. The golf club is in contact with the ball for 4.5×10^{-3} s.

Calculate the average force (in N) exerted by the club on the ball during the time they are in contact.

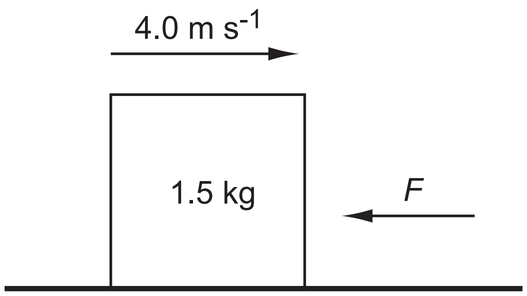
. . . .

Q41: A ripe tomato of mass 0.085 kg is thrown at a wall with velocity 5.4 m s $^{-1}$.

Calculate the magnitude of the impulse applied to the tomato by the wall, in kg m s $^{-1}$.

. . . .

Q42: An object of mass 1.5 kg is moving from left to right with velocity 4.0 m s -1 , as shown in the diagram. A force F = N acts on the object for 3.0 s, in the direction from right to left.



Bearing in mind the vector nature of momentum and force, calculate:

1. the impulse applied to the object, in kg m s $^{-1}$;
2. the new velocity of the object, in m s $^{-1}$.

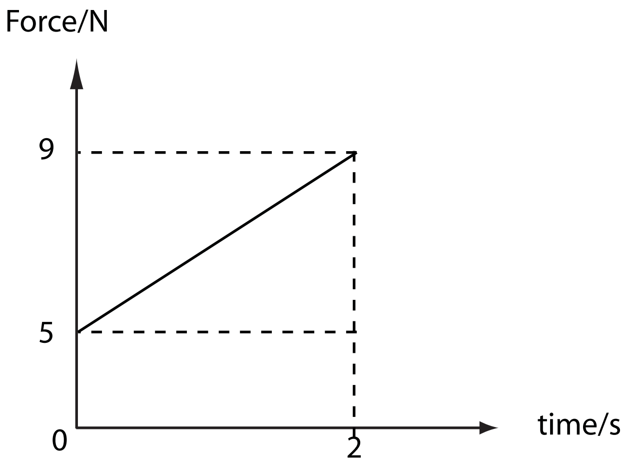
. . . .

Q43: Two objects A (mass 6.0 kg) and B (mass 2.0 kg) collide head-on on a linear air track. Before the collision, A is moving towards B at 3.6 m s $^{-1}$ and B is stationary. After the collision, A continues moving in the same direction at 1.6 m s $^{-1}$.

1. Calculate the velocity of B after the impact, in m s $^{-1}$;
2. Calculate the impulse applied to B, in kg m s $^{-1}$.

. . . .

Q44: The following graph shows how the force exerted on a mass of 0.8 kg varies with time. The mass is initially at rest.



1. Calculate the impulse on the mass.
2. Calculate the velocity with which the mass moves off.

The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Topic 4

Gravitation

Contents

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4.3 Summary 137
4.4 Extended information 137
4.5 Assessment 138

Learning objective

By the end of this section you should be able to:

- resolve the motion of a projectile with an initial velocity into horizontal and vertical components and their use in calculations;
- describe Newton's thought experiment.

In an earlier topic we learned about objects in freefall. In this topic we will concentrate on objects that have been projected. That is they have a horizontal component to their velocity as well as a vertical component.

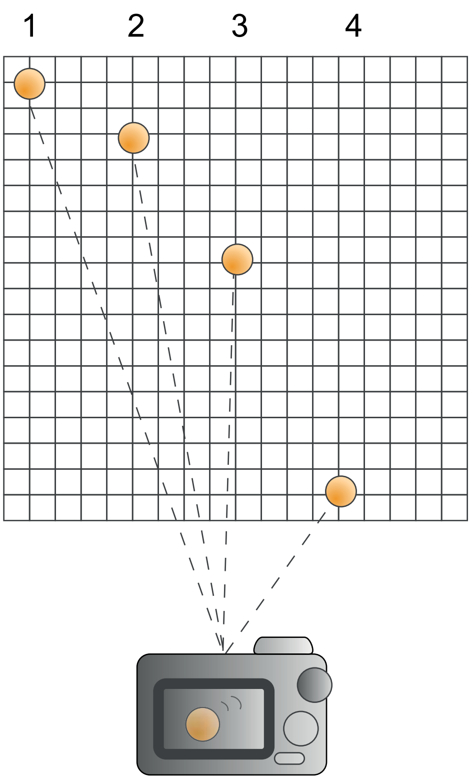
We will then see how *Newton's Thought Experiment* can be used to explain the motion of satellites around the Earth.

4.1 Projectiles

A projectile is an object that is flying through the air under the influence of gravity. This means that the object is moving in two dimensions.

If we use a video capture system to view a ball projected horizontally we obtain the following data.

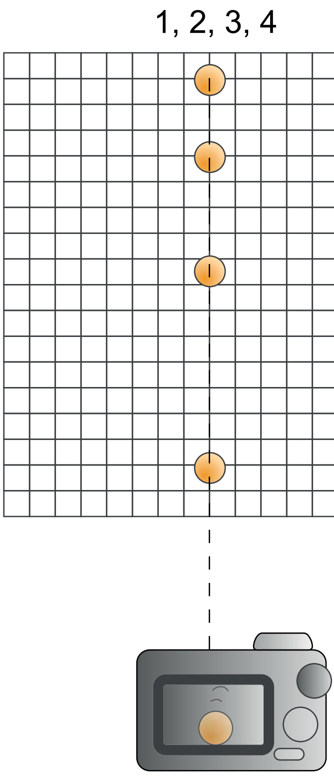
A video capture system takes a series of images of a moving object, usually against a grid, and a computer then displays all the images superimposed on one another so the motion can be analysed.



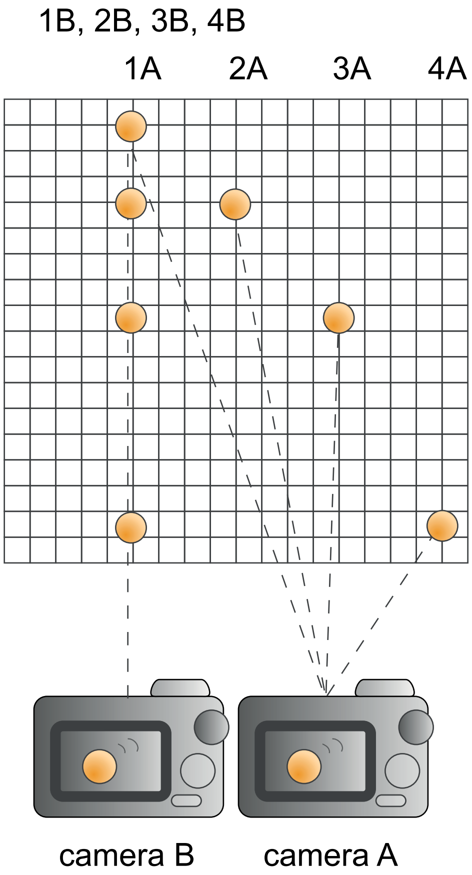
There are two things to note in particular.

1. In the horizontal direction the images are the same distance apart. As the time between each image is the same then in the horizontal direction the ball is moving at a constant speed. This is because in the horizontal direction there is no unbalanced force.
2. In the vertical direction the images are getting further apart. This means that the ball is accelerating in the vertical direction.

If we use the video capture system with the same settings to analyse a ball falling from rest we obtain the following data.



We can compare this with the projectile.



We can see that in the vertical direction the motion is identical. This is because a projectile is falling under the influence of gravity.

To solve problems involving motion of an object in two dimensions, we usually split the motion into vertical and horizontal components. If the only acceleration is the acceleration due to gravity g, then there is no horizontal acceleration of the object.

Examples of projectiles include any ball that has been thrown or hit and a bullet that has been fired from a gun.

All projectiles fly freely, that is to say they do not have an engine or a rocket to control their movement. This means aeroplanes are not projectiles.

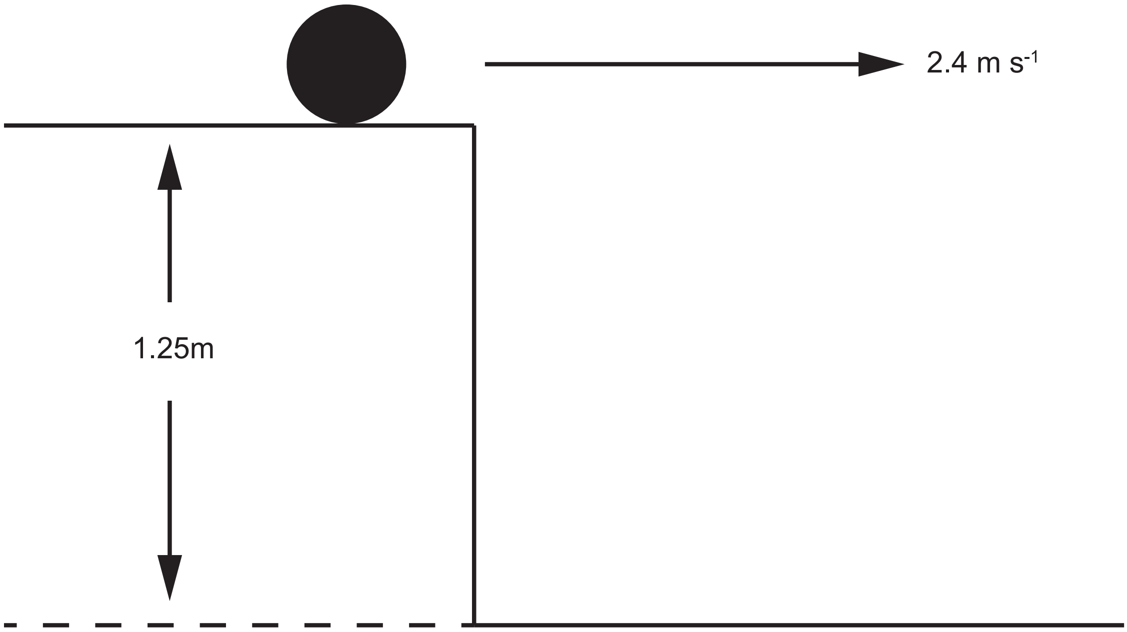
When analysing projectile motion we usually ignore the effects of air resistance so gliders and parachutists are not considered in this work.

You may wish to refer back to Unit 1 Topic 1 to remind yourself about the treatment of vectors.

You should make sure you know how to resolve a velocity vector into its horizontal and vertical components

Examples

1. A ball rolls over the edge of a horizontal surface as shown.



a) Calculate the time it will take to hit the floor.
b) Calculate the vertical component of the velocity as it hits the floor.
c) Calculate the resultant velocity, both magnitude and direction, as the ball hits the floor.

a) Solve this part of the question by considering the vertical motion. Since the ball is rolling horizontally the initial velocity is 0 m s^{-1}.

$$s = 1.25 \text{ m}$$
$$u = 0 \text{ m s}^{-1}$$
$$a = 9.8 \text{ m s}^{-2}$$
$$t = ?$$

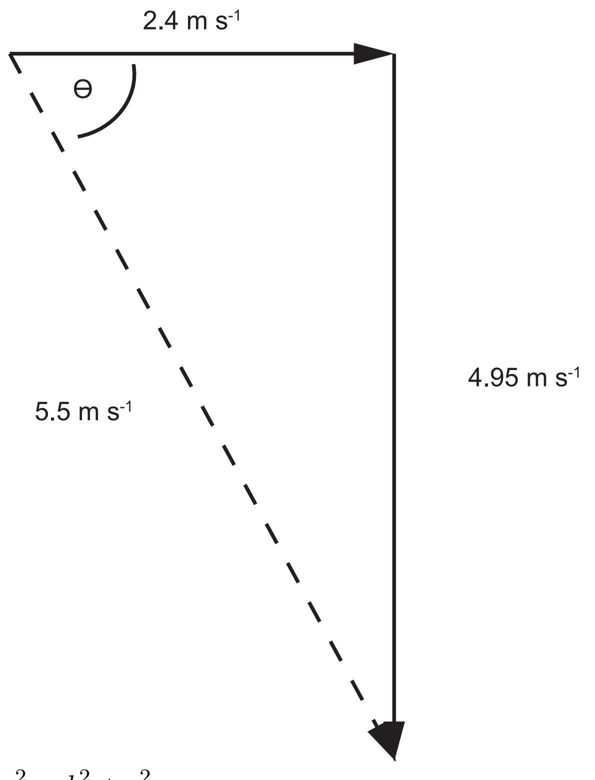
$$s = ut + \frac{1}{2}a\,t^2$$
$$1.25 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$
$$1.25 = 4.9 \times t^2$$
$$t^2 = \frac{1.25}{4.9}$$
$$t = \sqrt{0.255}$$
$$t = 0.505 \text{ s}$$

b) Again solve this part of the question by considering the vertical motion.

$$s = 1.25 \text{ m}$$
$$u = 0 \text{ m s}^{-1}$$
$$a = 9.8 \text{ m s}^{-2}$$
$$t = 0.505 \text{ s}$$
$$v = ?$$

$$v = u + at$$
$$v = 0 + 9.8 \times 0.505$$
$$v = 4.95 \text{ m s}^{-1} \text{ downwards}$$

c) The horizontal component of the velocity remains constant throughout the flight. The final vertical velocity is 4.95 m s^{-1}. In order to find the final resultant velocity draw a right angle triangle. Calculate the value of the hypotenuse which is the magnitude of the final velocity and the angle below the horizontal.

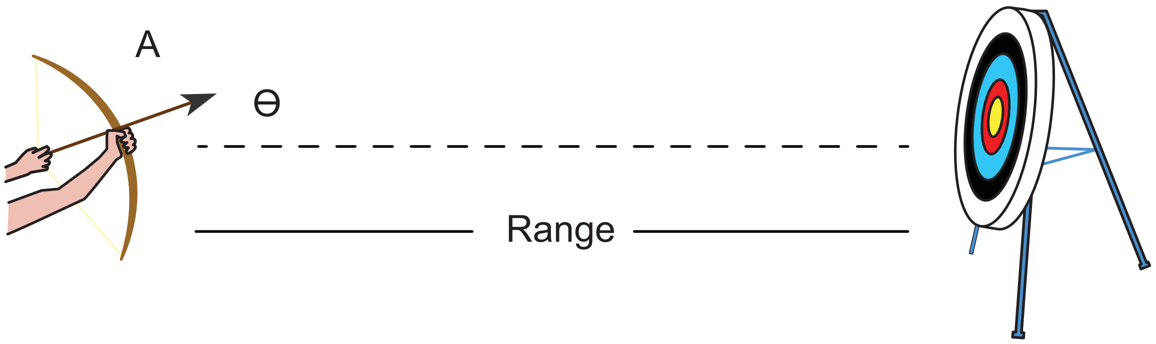


$$a^2 = b^2 + c^2$$
To find hypotenuse: $a^2 = 2.4^2 + 4.95^2$
$$a = 5.5 \text{ m s}^{-1}$$

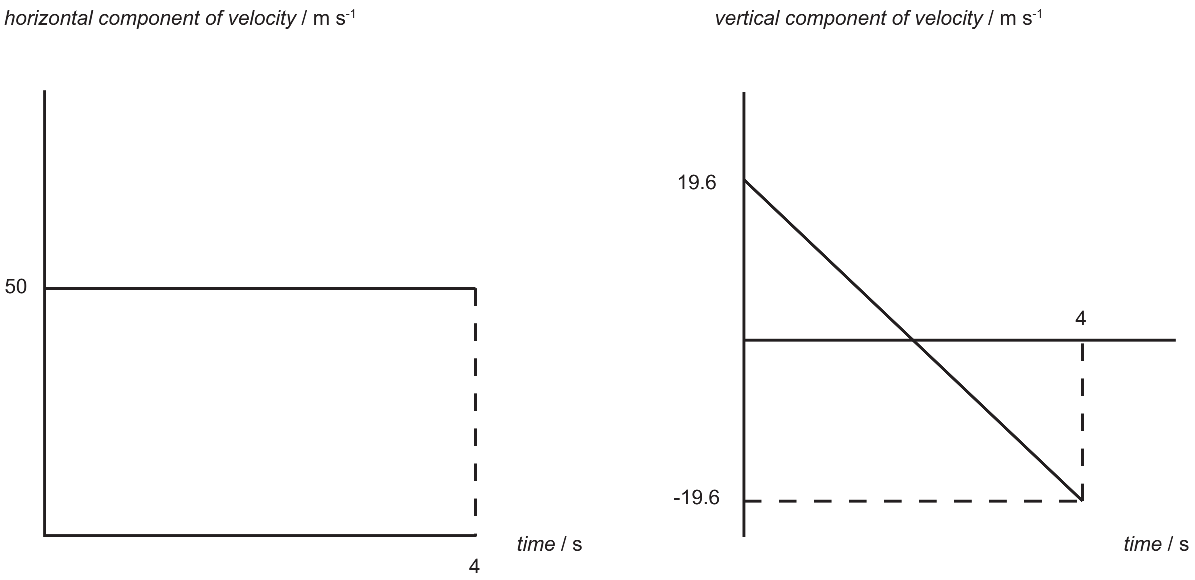
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
To find θ: $\tan\theta = \frac{4.95}{2.4}$
$$\theta = 67° \text{ below the horizontal}$$

. . . .

2. An arrow is fired with a velocity v from position A at an angle of θ to the horizontal. It hits the target after 4 seconds.

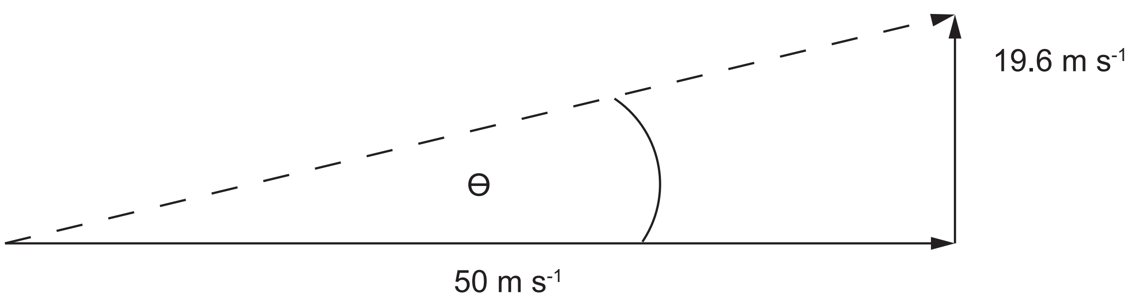


The horizontal and vertical components of its velocity are shown in the following graphs.



a) Calculate the angle at which the arrow is fired.
b) Calculate the range of the flight.
c) Calculate the maximum height of the arrow above the firing height.
d) The archer fires a second arrow with the same horizontal velocity but with a greater initial vertical component of velocity. Explain the effect this has on the position that the arrow hits the target.

a) In order to find the angle at which that arrow is fired draw a right angle triangle.



To find θ:
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan\theta = \frac{19.6}{50}$$
$$\theta = 21^\circ \text{ above the horizontal}$$

b) Range is the horizontal displacement so use horizontal information.

$$u_h = 50 \text{ m s}^{-1}$$
$$t = 4\ s$$
$$s = ?$$

$$s = vt$$
$$s = 50 \times 4$$
$$s = 200 \text{ m horizontally}$$

c) At maximum height the vertical component of velocity is 0 m s-1 so use vertical information.

Watch the acceleration is taken as negative because the initial velocity is upwards and the acceleration due to gravity is downwards. Also take care that u_v v_v = *0 m* s^{-1} rather than u_v = *m* s^{-1}.

$$u_v = 19.6 \text{ m s}^{-1}$$
$$v_v = 0 \text{ m s}^{-1}$$
$$a = -9.8 \text{ m s}^{-2}$$
$$s = ?$$

$$v^2 = u^2 + 2as$$
$$0^2 = 19.6^2 + 2 \times -9.8 \times s$$
$$0 = 394 - 19.6 \times s$$
$$s = 19.6 \text{ m upwards because the value is positive}$$

Part c) could also have been solved using $v_v = u_v + at$ where the time to reach maximum height is 2 seconds.

d) The second arrow has the same horizontal velocity therefore it will again take 4 seconds to reach the target. Since the initial vertical component of the velocity is greater it will take more than 2 seconds to reach maximum height. Therefore in 4 seconds it will not have fallen back to the level of release. This means that the arrow will hit the target above its centre.

....

3. At an athletics meeting, a long jumper takes off with a velocity of 7.8 m s^{-1} at an angle of 25° to the ground. How far does she jump?

To solve this problem, we must split the motion into horizontal and vertical components. Horizontally, her velocity is constant since there is no horizontal acceleration. We can use the simple relationship *displacement = velocity* $\times$ *time*, which in this case gives us

$$s_x = u\cos\theta \times t$$
$$\therefore s_x = 7.8 \times 0.9063 \times t$$
$$\therefore s_x = 7.069t$$

We can find the value of *t* by looking at the vertical motion, and finding the time from take-off until the jumper returns to the ground. That is to say, the time when her displacement s_y is zero again.

Her initial velocity upwards is *u* sinθ m s^{-1} and her acceleration upwards is -9.8 m s^{-2}.

$$s = ut + \frac{1}{2}at^2$$
$$\therefore 0 = (7.8\sin 25 \times t) + \left(\frac{1}{2} \times -9.8 \times t^2\right)$$
$$\therefore 0 = 3.296t - 4.9t^2$$
$$\therefore 4.9t^2 = 3.296t$$
$$\therefore 4.9t = 3.296$$
$$\therefore t = 0.673 \text{ s}$$

We can substitute this value of *t* into our equation for horizontal motion:

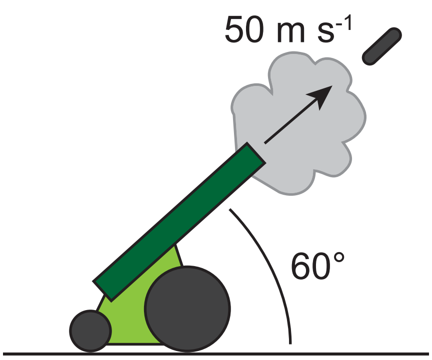
$$s_x = 7.069t$$
$$\therefore s_x = 7.069 \times 0.673$$
$$\therefore s_x = 4.8 \text{ m}$$

The long jumper therefore jumps a distance of 4.8 m.

. . . .

4.

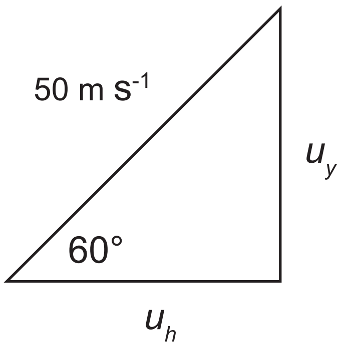
A shell is fired from a cannon as shown.



Calculate the shell's:

1. horizontal component of initial velocity;
2. vertical component of initial velocity;
3. maximum height.

Draw a right angle triangle to help find the components of the initial velocity.



1. u_h = v cos θ = 50 cos 60° = 25m s^{-1}
2. u_v = v sin θ = 50 sin 60° = 43m s^{-1}
3. At maximum height the vertical velocity will be zero.
   Since u_v is upwards and the acceleration due to gravity is downwards, make the acceleration due to gravity a negative value.

$v_v^2 = u_v^2 + 2as$

$0 = 43^2 + 2 \times (-9.8) \times s$

s = 94m

Projectile motion Go online

Q1: If a ball is projected horizontally at 10 m s^{-1} from a 5.0 m high wall, how far will it land from the base of the wall?

. . . .

Q2: Now instead of being projected horizontally let us project the ball at 30° to the horizontal. The initial speed of the ball is still 10 m s^{-1} and the wall is still 5 m high. What is the range now?

By considering the vertical and horizontal components separately, the kinematic relationships can be applied to the motion of a projectile.

4.2 Satellites

The time it takes a satellite to complete one orbit is known as its period.

The period of a satellite is determined by its height above the surface of the Earth. The higher the orbit the greater the period of the orbit.

Many people think that satellites just float in space, this is wrong. Satellites are held in an orbit because of the force of gravity that is acting on them. If a satellite does not have a large enough horizontal velocity it will be pulled closer to the surface of the Earth.

Communications satellites are often placed at a height of 35 000 km above the equator. This gives a satellite a period of 24 hours - the same time it takes the earth to turn once on its axis.

When viewed from the surface of the Earth they appear to be stationary. This is because they are turning through the same angle as the Earth in any given time.

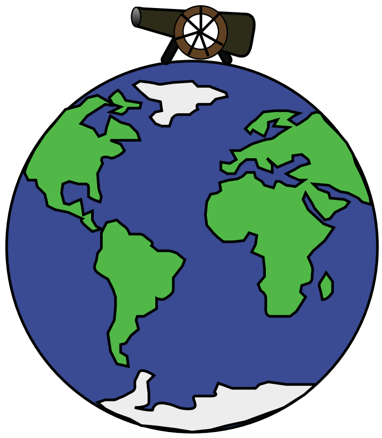
These satellites are called geostationary satellites because these satellites appear to be stationary with respect to the Earth dish aerials on the ground do not have to track their movements.

This is why television satellite dish aerials do not need to move to continuously receive a signal. It is also why these aerials (in the northern hemisphere) always point in a southerly direction towards the equator.

Mapping and survey satellites are placed in a lower orbit so they have a shorter period. These satellites are often placed in a pole to pole orbit rather than over the equator. These two factors allow them to pass over the entire surface of the earth after a period of time.

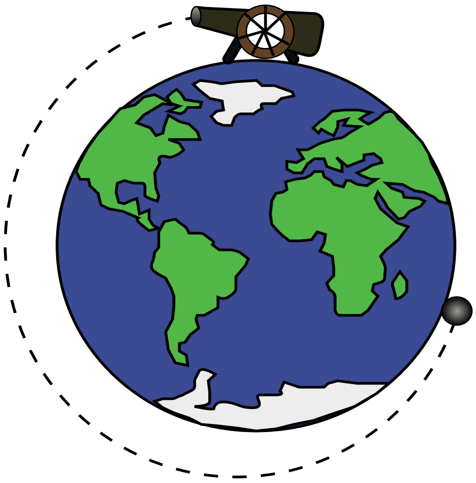
Newton's thought experiment Go online

Image an impossibly large cannon being used to fire a projectile at high speed.



When the cannon is fired the projectile falls towards the Earth following a curved path.

We can see that the projectile goes part of the way around the Earth before landing. If we increase the projectiles starting speed it will go a greater distance.



If we launch the projectile at exactly the correct speed it will continue around the Earth at the same height all the way. It will be in orbit.

Satellites stay in orbit because their curved projectile path matches the curve of the Earth so the satellite always stays the same distance above the surface of the Earth.

4.3 Summary

Summary

You should now be able to:

- Resolve the motion of a projectile with an initial velocity into horizontal and vertical components and their use in calculations.
- Describe Newton's thought experiment and use it to explain why satellites remain in orbit.

4.4 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

Links

- This site provides good revision materials on projectiles:
  http://www.physicsclassroom.com/class/vectors/u3l2a.cfm
- Another site covering projectiles:
  http://zonalandeducation.com/mstm/physics/mechanics/curvedMotion/projectileMotion/generalSolution/generalSolution.html
- This is an engaging simulation which allows the user to enter data, observe flight paths and calculate resulting values:
  http://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html
- This interesting site explores types of orbits:
  http://marine.rutgers.edu/mrs/education/class/paul/orbits2.html

4.5 Assessment

End of topic 4 test Go online

The following test contains questions covering the work from this topic.

The following data should be used when required:
Acceleration due to gravity g = 9.8 m s^{-2}

The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Q3: A ball rolls off the edge of a table. The table top is 0.75 m above the ground, and the ball strikes the ground a horizontal distance 1.7 m from the table.

1. Calculate the time taken in s for the ball to reach the ground from the moment it rolls off the table.
2. Calculate the initial (horizontal) velocity of the ball (in m s^{-1}) as it rolled along the table top.

. . . .

Q4: An artillery shell is fired from a launcher. The shell is fired at an angle 30° to the ground, with an initial velocity of 260 m s^{-1}.

Calculate the horizontal range of the shell, in m.

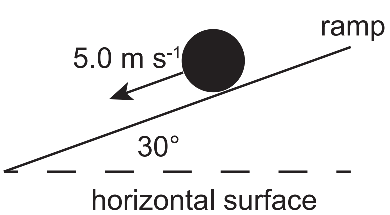
. . . .

Q5: Charlie throws a small toolkit to his colleague Bob who is working on the roof of a building. Charlie throws the toolkit with initial velocity 12 m s^{-1}, at an angle 62° to the ground. Bob catches the toolkit at the instant when it is travelling horizontally.

1. Calculate the height of the building, in m.
2. Calculate how far Charlie is standing from the foot of the building, in m.

. . . .

Q6: A ball rolls down a ramp which is at an angle of 30° to the horizontal. It is travelling at 5 m s^{-1} when it leaves the end of the ramp, as shown in the diagram.



The ball reaches the horizontal surface after 0.50 s.

1. Calculate the height of the end of the ramp above the horizontal surface.
2. Calculate the horizontal distance travelled by the ball before it hits the horizontal surface.

Topic 5

Gravity and mass

Contents

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Learning objective

By the end of this section you should be able to:

- describe and use the concept of weight as the effect of a gravitational field on a mass;
- state and apply Newton's Universal Law of Gravitation;
- calculate the weight of an object and the acceleration due to gravity.

Previously we examined what happened to objects as they fell near the earth's surface. We used this knowledge to think about the motion of projectiles and then went on to examine satellites.

In this topic we will extend this to think about how objects are affected by a gravitational field, including fields other than the Earth's field. We will end by examining Newton's universal law of gravity.

In this part of the topic we will be investigating gravity and the gravitational force that exists between any two objects. As well as calculating the force, the concept of a gravitational field will be introduced.

Throughout this topic, some approximations will be made with regard to planets and their orbits. In all the examples and exercises in this topic, it will be assumed that the Sun, the Moon, the Earth and other planets are all spherical objects. Furthermore, we will be treating their orbits as circular.

The force of gravity is relatively very weak compared to electromagnetic forces but it does act over very, very large distances.

5.1 Gravity

Gravity is responsible for the formation of the solar system.

At some point, generally believed to be between 4 and 5 billion years ago, a huge cloud of molecules collapsed together due to their own gravitational attraction. Most of these molecules collapsed into a gravitational centre forming the Sun. However a small amount of mass formed a disk that circled the newly formed star. The gravitational attraction between the particles that made up this disk then resulted in the formation of the planets we can observe today.

Gravitational forces are always attractive, they are never repulsive. This means any two masses will attract each other due to the gravitational force between them.

This process is known as the aggregation of matter.

When a star forms the matter making it up becomes more and more compressed under the influence of gravity. This process causes the temperature of the matter to rise until the nuclear material ignites a fusion reaction. This fusion reaction creates a pressure which causes an outward expanding force. When the gravitational force and the thermal force are equal but in opposite direction the star reaches a temporary equilibrium. It should be noted that this temporary equilibrium is likely to last for many billions of years.

Once the star has used up most of its material in the fusion process the thermal force ceases to overcome the gravitational force and the star undergoes a gravitational collapse.

This is often called the death of a star.

The final state of a star depends on several factors such as how much material was present at its formation. The normal fates of most stars are either White dwarfs or Neutron stars.

Some stars undergo such an extreme gravitational collapse that they form a black hole. When this happens the gravitational field around the star becomes so great that not even light has a high enough velocity to escape from the collapsed star.

As we know the Earth follows a curved path around the Sun. This means that, according to Newton's first law of motion, there must be an unbalanced force acting on the Earth. This force is provided by the gravitational pull of the Sun. The same is true for the orbit of the Moon. The Earth's gravity is exerting a force on the Moon.

Gravity has also been used to assist exploration in the solar system. During the 1990s NASA sent a probe called Galileo to investigate the planet Jupiter. They found that it would be very difficult to give the probe enough energy to reach Jupiter directly so they devised an ingenious method of boosting the probe's speed during its flight. This was done by flying the probe past Venus and using the planet's gravitational field to sling shot it with increased speed. The process was then repeated using the Earth's gravitational field (twice) before the probe had enough speed to reach Jupiter.

5.2 Mass, weight and gravitational field strength

As we have already seen, the mass of an object is the quantity of matter that the object contains.

The **weight** of an object on Earth is the pull of the Earth on the object. Weight is a force so it is measured in newtons (N).

The force that acts on an object because of its mass is the force of gravity or the weight of the object. The force of gravity near the Earth's surface gives all objects the same acceleration (if the effects of friction can be ignored). The approximate value of the acceleration due to gravity near to the Earth's surface is 9.8 m s^{-2}.

More precisely, the region of space around an object such as the Earth, in which the Earth exerts a gravitational force on another object placed in that region is called the **gravitational field** of the Earth. The concept of a field is used in many situations in physics, such as the electric field surrounding a charged particle, or the magnetic field around a bar magnet.

The **gravitational field strength** is the ratio of weight to mass, or the weight per unit mass.

$$\begin{aligned} \text{weight} &= \text{mass} \times \text{gravitational field strength} \\ W &= mg \end{aligned} \tag{5.1}$$

....

The symbol for gravitational field strength is *g*, and it is measured in newtons per kilogram (N kg^{-1}).

Example : Weight and mass

Calculate the weight of a person of mass 70.0 kg.

Using *W* = *mg*, we have

$$\begin{aligned} W &= mg \\ \therefore W &= 70.0 \times 9.8 \\ \therefore W &= 686 \text{ N} \end{aligned}$$

The weight of the object is therefore 686 N.

The gravitational field strength at the surface of the planets, and the Sun, has different values. Because of this, for any object, the mass remains constant but the weight can vary, since weight depends on the gravitational field strength.

The approximate value of the gravitational field strength at the surface of the planets, and the Sun, is as shown in the table.

Table 5.1: Gravitational field strengths

Body in Solar System	**Gravitational field strength** ($N\ kg^{-1}$)
Earth	9.8
Jupiter	26
Mars	4
Mercury	4
Moon	1.6
Neptune	12
Saturn	11
Sun	270
Venus	9

Example : Mars buggy

A Mars buggy has a mass of 120 kg.

Calculate its weight on the Earth and on Mars.

Using *W* = *mg* and the information contained in Table 5.1 we have

$$W = mg$$

on Earth, $W = 120 \times 9.8 = 1176$ N

on Mars, $W = 120 \times 4 = 480$ N

The weight of the buggy is 1176 N on Earth but 480 N on Mars.

Notice that the mass on Mars is the same as the mass on Earth.

Quiz: Mass, weight and gravitational field strength Go online

Q1: What is the equation that links mass, weight and gravitational field strength?

a) *W* = *m* x *g*
b) *m* = *W* x *g*
c) *W* = m^2 x *g*
d) *m* = W^2 x *g*
e) *W* = $(m \times g)^2$

....

Q2: What is the weight of an object of mass 25.0 kg in a gravitational field strength of 9.8 N kg^{-1}?

a) 60100 N
b) 1.6 N
c) 6130 N
d) 2.55 N
e) 245 N

. . . .

Q3: Which statement about the mass and the weight of an object is true?

a) Both the mass and the weight of the object are measured in the same units.
b) Only the mass of the object is affected by the gravitational field strength.
c) Only the weight of the object is affected by the gravitational field strength.
d) Both the mass and the weight of the object are affected by the gravitational field strength.
e) Both the mass and the weight of the object are independent of the gravitational field strength.

. . . .

Q4: A probe of mass 160 kg lands on Mars, where the gravitational field strength is 4 N kg^{-1}.

What is the weight of the probe on Mars?

a) 16.3 N
b) 40.0 N
c) 160 N
d) 640 N
e) 1570 N

. . . .

Q5: An object falls in a uniform gravitational field with air resistance and reaches its terminal velocity.

From the instant the object is released, the acceleration of the object during its fall

a) remains constant.
b) increases then decreases to zero.
c) remains constant for a period of time then decreases to zero.
d) increases from zero uniformly.
e) decreases to zero.

5.3 Newton's universal law of gravitation

Working in the 17th century, Sir Isaac Newton discovered the **universal law of gravitation**. He used his own observations, along with those of Johannes Kepler, who had formulated a set of laws that described the motion of the planets around the Sun.

Newton's law of gravitation states that there is a force of attraction between any two objects in the universe. The size of the force is proportional to the product of the masses of the two objects, and inversely proportional to the square of the distance between them. This law can be summed up in the equation

$$F = \frac{Gm_1m_2}{r^2} \tag{5.2}$$

In Equation 5.2, m_1 and m_2 are the masses of the two objects, and r is the distance between them. The constant of proportionality is G, the gravitational constant. The value of G is 6.67 $\times$ 10^{-11} m^3 kg^{-1} s^{-2}. A simple example will show us the order of magnitude of this force.

Example

Consider two point masses, each 2.00 kg, placed 1.20 m apart on a table top. Calculate the magnitude of the gravitational force between the two masses.

Using Newton's law of gravitation

$$\begin{aligned}
F &= \frac{Gm_1m_2}{r^2} \\
\therefore F &= \frac{6.67 \times 10^{-11} \times 2.00 \times 2.00}{1.20^2} \\
\therefore F &= \frac{2.668 \times 10^{-10}}{1.44} \\
\therefore F &= 1.85 \times 10^{-10} \text{ N}
\end{aligned}$$

The gravitational force between these two masses is only 1.85 $\times$ 10^{-10} N. This is an extremely small force, one which is not going to be noticeable in everyday life.

We rarely notice the gravitational force that exists between everyday objects as it is such a small force. You do not have to fight against gravity every time you walk past a large building, for example, as the gravitational force that the building exerts on you is too small to notice. Because the constant G in Newton's Law of Gravitation is so small, the gravitational force between everyday objects is usually negligible. The force only really becomes important when we are dealing with extremely large masses such as planets.

The gravitational force is always attractive, and always acts in the direction of the straight line joining the two objects. According to Newton's third law of motion, the gravitational force is exerted on **both** objects. As the Earth exerts a gravitational force on you, so you exert an equal force on the Earth.

Most of the work we will be doing on gravitation concerns the forces acting between planets and stars. So far we have only considered point objects, so do we need to adapt Newton's Law of Gravitation when we are dealing with larger bodies? The answer is no - for spherical objects (or more accurately, objects with a spherically symmetric mass distribution), the gravitational interaction is exactly the same as it would be if all the mass was concentrated at the centre of the sphere. Remember, we will assume in all our calculations that the planets and stars we are dealing with are spherical.

Example

The Earth has a radius of 6.38 $\times$ 10^6 m and a mass 5.97 $\times$ 10^{24} kg. What is the gravitational force due to the Earth acting on a woman of mass 60.0 kg standing on the surface of the Earth?

The solution is obtained by calculating the force between two point objects placed 6.38 $\times$ 10^6 m apart, if we treat the Earth as a uniform sphere. Hence

$$F = \frac{Gm_1m_2}{r^2}$$

$$\therefore F = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 60.0}{(6.38 \times 10^6)^2}$$

$$F = \frac{2.389 \times 10^{16}}{4.070 \times 10^{13}}$$

$$F = 587 \text{ N}$$

The gravitational force acting on the woman is 587 N, and this force is directed towards the centre of the Earth.

If you calculate the force due to gravity acting on the woman by another method, using F=m $\times$ g, you should find that you obtain a very similar answer, 588 N. The slight difference comes from using a value of g = 9.8 m s^{-2}. As we shall see later in this topic, the value of g obtained by assuming the Earth to be a sphere of uniform density is slightly less than this value.

5.4 Weight

The **weight** W of an object of mass m can be defined as the gravitational force exerted on it by the Earth.

$$W = F_{grav} = \frac{Gm_E m}{{r_E}^2} \tag{5.3}$$

....

In Equation 5.3, m_E and r_E are the mass and radius of the Earth.

The acceleration due to gravity, g, is found from Newton's second law of motion

$$F = ma$$
$$\therefore W = mg$$

We can substitute for F in this equation

$$mg = \frac{Gm_E m}{{r_E}^2}$$
$$\therefore g = \frac{Gm_E}{{r_E}^2} \tag{5.4}$$

So the acceleration of an object due to gravity close to the Earth's surface does not depend on the mass of the object. In the absence of friction, all objects fall with the same acceleration.

Equation 5.4 is a specific equation for calculating g on the Earth's surface. In general, at a distance r from the centre of a body (a star or a planet, say) of mass m, the value of g is given by

$$g = \frac{Gm}{r^2} \tag{5.5}$$

The mass of an object is constant; it is an intrinsic property of that object. The weight of an object tells us the magnitude of the gravitational force acting upon it, so it is not a constant.

Example

Compare the values of g on the surfaces of the Earth (m_E = 5.97 × 10^{24} kg, r_E = 6.38 × 10^6 m) and the Moon (m_M = 7.35 × 10^{22} kg, r_M = 1.74 × 10^6 m).

To solve this problem, use Equation 5.5 with the appropriate values

$$g = \frac{Gm_E}{{r_E}^2}$$
$$\therefore g = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{\left(6.38 \times 10^6\right)^2}$$
$$\therefore g = 9.78 \text{ m s}^{-2}$$

$$g = \frac{Gm_M}{{r_M}^2}$$
$$\therefore g = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{\left(1.74 \times 10^6\right)^2}$$
$$\therefore g = 1.62 \text{ m s}^{-2}$$

The value for g on the surface of the Earth is 9.78 m s^{-2}, whilst on the surface of the Moon the value of g is 1.62 m s^{-2}.

This means that while the mass of an object at the surface of the Earth and the surface of the Moon will be the same, the weight at the surface of the Earth will be much greater than at the surface of the Moon.

Acceleration due to gravity Go online

Q6: A mass (e.g. a ball) is dropped from 100m on different planets on the solar system and the height at time intervals is compared in side by side boxes.

The height is given by the formula h = 100 - (0.5 × g × t^2)

Complete the table to give the height of the mass at the times in the table. The values of 'g' on each of the planets is given below.

- Earth 9.8 N kg^{-1}
- Jupiter 26 N kg^{-1}
- Mars 4.0 N kg^{-1}
- Mercury 4.0 N kg^{-1}
- Neptune 12 N kg^{-1}
- Saturn 11 N kg^{-1}
- Venus 9.0 N kg^{-1}

Remember that if the height reaches zero the mass cannot fall any further.

time /s	Earth	Jupiter	Mars	Mercury	Neptune	Saturn	Venus
0	100	100	100	100	100	100	100
1							
2							
3							
4							
5							
6							
7							
8							

Quiz: Gravitational force Go online

Useful data:

Gravitational constant G	6.67×10^{-11} Nm^3 kg^{-1}s^{-2}
Mass of the Moon m_M	7.35×10^{22} kg
Radius of the Moon r_M	1.74×10^{6} m
Mass of Venus m_V	4.87×10^{24} kg
Radius of Venus r_V	6.05×10^{6} m

Q7: Two snooker balls, each of mass 0.25 kg, are at rest on a snooker table with their centres 0.20 m apart. What is the magnitude of the gravitational force that exists between them?

a) 2.1×10^{-11} N
b) 4.3×10^{-11} N
c) 8.3×10^{-11} N
d) 1.0×10^{-10} N
e) 4.2×10^{-10} N

....

Q8: The Sun exerts a gravitational force F_S on the Earth. The Earth exerts a gravitational force F_E on the Sun. Which one of these statements about the magnitudes of F_S and F_E is true?

a) $F_S = F_E$
b) $F_S < F_E$
c) $F_S > F_E$
d) $\frac{F_S}{F_E} = \frac{mass_S}{mass_E}$
e) $\frac{F_S}{F_E} = \frac{(mass_S)^2}{(mass_E)^2}$

....

Q9: What is the weight of a 5.00 kg mass placed on the surface of the Moon?

a) 0.123 N
b) 1.42 N
c) 1.62 N
d) 8.10 N
e) 40.5 N

. . . .

Q10: An object is taken from sea level to the top of Mount Everest. Which one of the following statements is true?

a) Its mass remains constant but its weight increases.
b) Its mass remains constant but its weight decreases.
c) Its weight remains constant but its mass increases.
d) Its weight remains constant but its mass decreases.
e) Neither its mass nor its weight alters.

. . . .

Q11: What is the value of the acceleration due to gravity on the surface of the planet Venus?

a) 0.887 m s^{-2}
b) 5.37 m s^{-2}
c) 6.67 m s^{-2}
d) 8.29 m s^{-2}
e) 8.87 m s^{-2}

5.5 Summary

Summary

You should now be able to:

- describe and use the concept of weight as the effect of a gravitational field on a mass.
- state and apply Newton's Universal Law of Gravitation.
- calculate the weight of an object and the acceleration due to gravity.

5.6 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

Links

- Although this site can be slow to load it provides a good explanation of weight: **http://www.av8n.com/physics/weight.htm**
- This is a simple NASA page on weight: **http://www.grc.nasa.gov/WWW/K-12/airplane/weight1.html**
- An excellent interactive website. Move the sun, earth, moon and space station to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it: **http://phet.colorado.edu/en/simulation/gravity-and-orbits**
- An excellent interactive website on Newton's Law of Gravitation. Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force: **http://phet.colorado.edu/en/simulation/gravity-force-lab**

5.7 Assessment

End of topic 5 test Go online

The following test contains questions covering the work from this topic.

The following data should be used when required:

universal constant of gravitation G	$6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$
mass of the Earth	5.97×10^{24} kg
mass of the Moon	7.35×10^{22} kg
mass of the Sun	1.99×10^{30} kg
radius of the Earth	6.38×10^{6} m
radius of the Moon	1.74×10^{6} m

The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Q12: A distant planet has mass 5.45×10^{25} kg. A moon, mass 2.54×10^{22} kg orbits this planet with an orbit radius of 7.96×10^{8} m.

Calculate the size of the gravitational force that exists between the moon and the planet, in N.

. . . .

Q13: Two identical solid spheres each have mass 0.603 kg and diameter 0.325 m

Find the gravitational force, in N, between them when they are touching.

. . . .

Q14: Given that the mass of the planet Neptune is 1.03×10^{26} kg and its radius is 2.48×10^{7} m, calculate the weight, in N, of a 6.64 kg mass on the surface of Neptune.

. . . .

Q15: The value of the acceleration due to gravity is not constant, decreasing with height above the surface of the Earth.

What is the value of the acceleration due to gravity, in m s $^{-2}$, in the ionosphere at a height 1.23×10^{5} m above the Earth's surface?

. . . .

Q16: On the surface of the Earth, a particular object has a weight of 17 N.

Calculate its weight on the surface of the Moon, in N.

. . . .

Q17: A planet in a distant galaxy has mass 7.07×0^{25} kg and radius 3.02×10^{7} m.

Calculate the value of the gravitational field strength on the surface of this planet, in N kg $^{-1}$

. . . .

Q18: The gravitational field strength at a distance 2.48×10^{6} m from the centre of a planet is 6.85 N kg $^{-1}$

Calculate the field strength at a distance 9.92×10^{6} m from the centre of the planet, in N kg $^{-1}$. (Note: it is possible to solve this problem without having to calculate the mass of the planet.)

. . . .

Q19: In a distant solar system, a planet (mass 3.42×10^{28} kg) is orbiting the sun (mass 5.81×10^{30} kg) with an orbit radius of 4.44×10^{11} m.

Calculate the magnitude of the net gravitational field strength midway between the planet and the sun, in N kg $^{-1}$

Topic 6

Special relativity

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Learning objective

By the end of this section you should be able to:

- describe what is meant by a frame of reference;
- explain why during the 19th century experimental evidence and theoretical consideration gave rise to a challenge to Newton's idea of absolute space;
- explain why the constancy of the speed of light led Einstein to postulate that space and time for a moving object are changed relative to a stationary observer;
- carry out calculations for time dilation and length contraction.

In this topic we will introduce Einstein's theory of special relativity.

We will do this by thinking about what is meant by a frame of reference and about some of the advances made in the 19th century which Einstein went on to explain.

We will also consider two of the implications of the theory of special relativity; time dilation and length contraction.

6.1 Frames of reference

We make all measurements within a frame of reference.

What do we use as a frame of reference?

When we say a car is moving at 12 m s^{-1} we mean to say that it is moving at 12 m s^{-1} relative to its surroundings e.g. the surface of the earth. This is called a frame of reference. Normally we assume we are stationary when we describe motion but this is not always the case.

Imagine you are on a train travelling at 30 m s^{-1} relative to the earth. If someone on the train is walking towards you at 2m s^{-1} their relative velocity to you is 2 m s^{-1}. To someone standing stationary relative to the earth their velocity would appear to be 28 m s^{-1}. We can see that the velocity of an object must be measured relative to a frame of reference.

To try and produce a set of laws for the universe Newton proposes that there is such a thing as absolute space. In other words there is an absolute frame of reference - space. Newton says that the space is unchanging and can therefore be used as a frame of reference.

What Newton is proposing is that the laws of physics are the same for an observer whether they are moving with a steady motion or they are at rest. This is also known as **Galilean invariance** after the Italian physicist and mathematician Galileo.

This assumption leads to some conclusions about how objects will behave in this absolute space. For example if a pair of masses, connected by a string, are set spinning in deep space the string will become taught due to absolute space. This may seem to be unimportant but it was a question that bothered many theoretical physicists over many years and was the centre of a heated debate that led to some startling conclusions.

6.2 Opinions on Newton's ideas

Mach (and others) disagrees about absolute space. Mach suggests that the spinning string becomes taut due to the presence of mass elsewhere in the universe. If there was no other mass in the universe, the string would not become taut. For Mach there is no such thing as absolute space. This is called a relationist view.

In addition to this, and other theoretical considerations about space, others who were working on the properties of electromagnetic radiations, including light, made discoveries that would affect the way we think about absolute space.

Clerk-Maxwell's theoretical work suggests that electromagnetic waves such as light have a specific velocity in a vacuum irrespective of the velocity of the object producing them.

Michelson and Morely conduct experiments that show the speed of light in a vacuum is the same for all observers irrespective of their relative velocity to the source of light.

Both of these discoveries seem to be incompatible with the idea of absolute space. If you are moving towards a source of light in absolute space then the light should have a higher velocity than if you are stationary with respect to absolute space. However both measurements and other theoretical considerations show that the speed of light is constant irrespective of the relative motion of the source and the observer.

A new theory about space and time is needed to explain these observations.

6.3 Special relativity

Einstein tried to reconcile these measurements and views. Einstein said that space-time taken together is absolute but space and time are individually relative. This gives rise to the special theory of relativity.

Einstein made the following two points:

1. The speed of light is absolute. It is always the same for all observers irrespective of their relative velocities.
2. The laws of Physics are the same for all observers inside their frame of reference.

These observations lead to some conclusions that at first seem strange but that have been confirmed by observation.

For two observers in the same frame of reference with identical accurate clocks the rate at which time elapses on these clocks will depend on their relative motion. If one observer is stationary in that frame of reference and the other is moving, the clock of the moving observer appears to be going slower than the stationary observer. Neither clock can be said to be wrong. If either clock is used to measure the speed of light for that observer they will arrive at the correct value of 3×10^8 m s-1. This is referred to as time dilation.

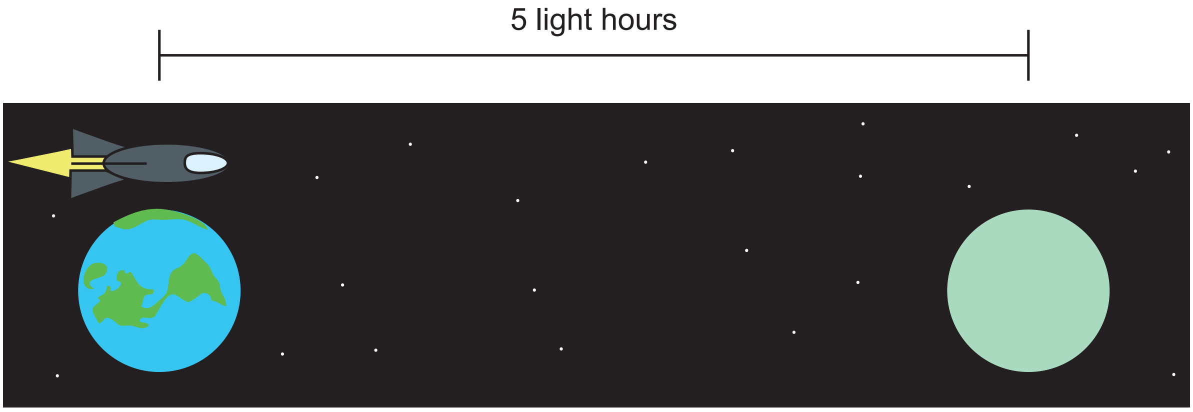
6.3.1 Time dilation

Time dilation Go online

At this stage there is an online activity. If however you do not have access to the internet you should read the explanation and ensure that you understand it.

A spaceship is flying a distance of 5 light hours, for example from Earth to the planet Pluto.

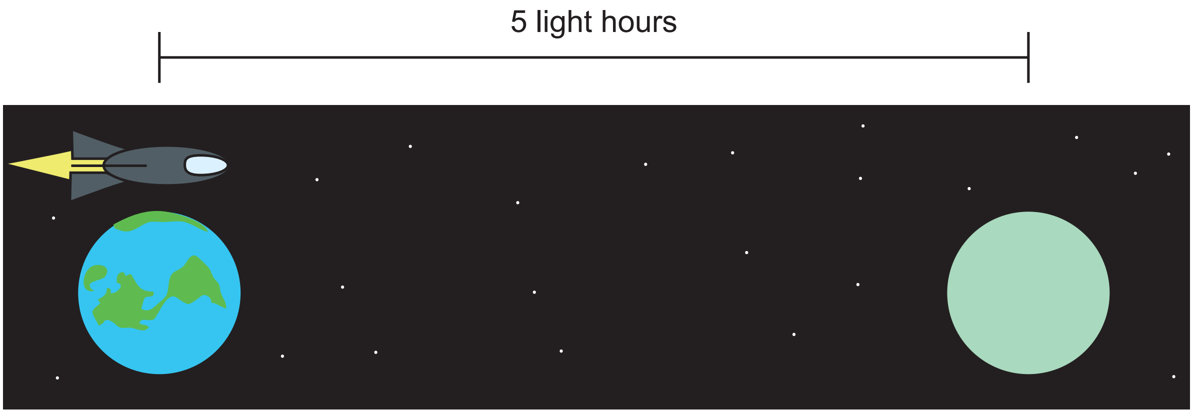
The diagram shows the elapsed time on the two planets (which are taken to be stationary in this frame of reference) and the time elapsed on the spaceship. At first the spaceship makes the journey at 0.01 % of the speed of light.



Speed: 0.000100c

Flying time (Earth system): 50000 hours
Flying time (Spaceship system): 49999.99975 hours

We can see that the clocks on the planets and the spaceship are almost the same - there is little time dilation at this speed. If the spaceship now makes the journey at 40 % the speed of light the effect of time dilation is significant.



Speed: 0.400000c

Flying time (Earth system): 12.5 hours
Flying time (Spaceship system): 11.456439 hours

We can see that the time elapsed on the spaceships clock is significantly less than the clocks on the two planets.

Here is an explanation of the phenomenon.

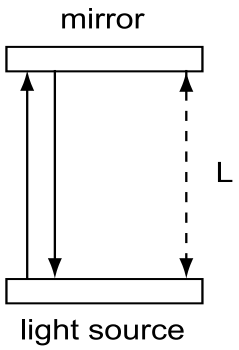
A glass sided spacecraft is travelling at 1.5 X 10^6 m s^{-1}. The height of the spacecraft is 30 m high, a light source is in the floor and a mirror directly above on the roof. There are two observers, one on the moving spacecraft and the other on the stationary space station. They both measure the time taken for a beam of light to travel from the floor to the mirror and back. The observer on the spacecraft records a time of 2.00 X 10^{-7} s. The observer on the space station records 2.01×10^{-7} s. This is because when both observers calculate the speed of light they must obtain the value of 3×10^8 m s^{-1}.

For the observer on the spacecraft the light has travelled 60 m but for the observer on the space station it has travelled further.

Let us now examine the relationship between t, the time measured by the observer on the moving spacecraft, and time t', (t dash) the time measured by the observer on the stationary space station.

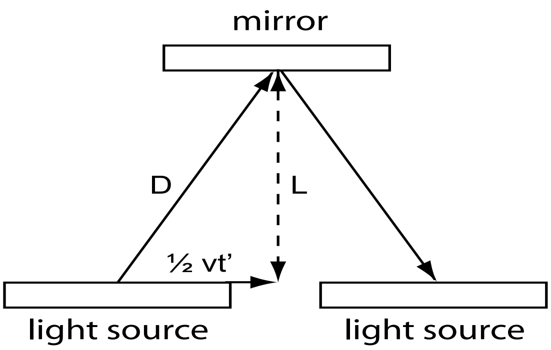
Proof of time dilation based on the activity

For the observer on the moving spacecraft the light appears to follow the following path.



The time taken for the observer on the moving spacecraft is **t** = **2L/c** where L is the distance from the floor to the mirror and c is the speed of light.

For an observer on the stationary space station the light appears to take the following path. This is because as the light moves up towards the mirror and reflects back down to the light source, the light source will have moved to the right due to the movement of the spacecraft.



The time for the light to travel from the source to the mirror and back to the source is measured by the observer on the stationary space station as **t'**= **2D/c**where D is the hypotenuse of the triangle described by the ray of light. The length of the horizontal side is equal to $^{1}/_{2}$ vt' where v is the velocity of the spacecraft.

Using Pythagoras theorem

$$D^2 = \left(\tfrac{1}{2}vt'\right)^2 + L^2$$

$$\text{so } \left(\frac{t'c}{2}\right)^2 = \left(\tfrac{1}{2}vt'\right)^2 + \left(\frac{tc}{2}\right)^2$$

$$\text{then } t'^2c^2 = v^2t'^2 + t^2c^2$$

$$t'^2\left(c^2 - v^2\right) = t^2c^2$$

$$t'^2 = \frac{t^2c^2}{(c^2 - v^2)}$$

$$t' = \sqrt{\frac{t^2c^2}{(c^2 - v^2)}}$$

so (dividing top and bottom under root sign by c^2)

$$t' = \frac{t}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$t' = \frac{t}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \qquad (6.1)$$

. . . .

Actual observation using very accurate atomic clocks show that clocks placed on fast moving aircraft show a smaller amount of elapsed time than identical clocks that have been kept stationary.

It should be noted that for speeds lower than 10% the speed of light the time dilation effect is extremely small.

Example Calculate the time dilation for a stationary observer relative to an observer travelling at 100000 m s^{-1} who measures the time elapsed as 24 hours.

Time elapsed for moving observer, t = 24 $\times$ 60 $\times$ 60 = 86400 s.

Time for stationary observer = t'

$$t' = \frac{t}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$t' = \frac{86400}{\sqrt{\left(1 - \left(\frac{100000}{3\times10^8}\right)^2\right)}}$$

$t' = 86400.0048$ s

Thus we can see the time dilation is extremely small even for what appears to be a high velocity.

If we repeat the calculation for an object travelling at 2×10^8 m s^{-1} we can see the difference.

Time elapsed for moving observer, t = 24 $\times$ 60 $\times$ 60 = 86400 s.

Time for stationary observer = t'

$$t' = \frac{t}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$t' = \frac{86400}{\sqrt{\left(1 - \left(\frac{2\times10^8}{3\times10^8}\right)^2\right)}}$$

$t' =$ 115918 s

The time elapsed for the stationary observer is 115918 s.

Measurements show that time elapses at different rates for different observers depending on their speed within a frame of reference.

6.3.2 Length contraction

When a stationary observer examines the length of a moving object they find that it is not as long as the length when stationary. This is a relativistic effect. Because time does not elapse equally for all observers when the time for an object to pass is measured by a stationary observer longer time is obtained than for an observer on the object. When the length of the object is then calculated from this value it will be found to be less than the length when the object is stationary. This is called length contraction. Length contraction is only significant at velocities very close to the velocity of light.

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{6.2}$$

. . . .

l is the length measured when the object is at rest, sometimes called l_0 (l nought), rest length.

l' is the length of the moving object measured by the stationary observer Notice l' is always less than l.

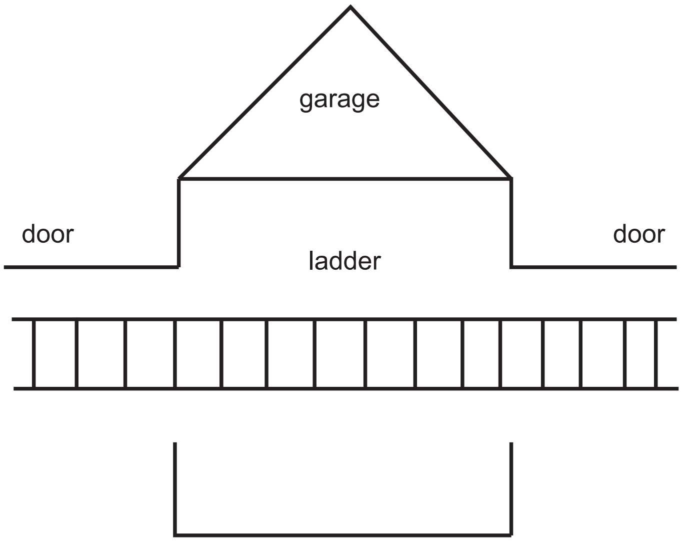
As with time dilation the effects are only noticeable at very high velocities.

Ladder Paradox

The ladder paradox is a thought experiment in special relativity. It is a useful example of the effect of length contraction depending on the relative motion of the observer. What appears to happen depends on the motion of the observer relative to the moving object.

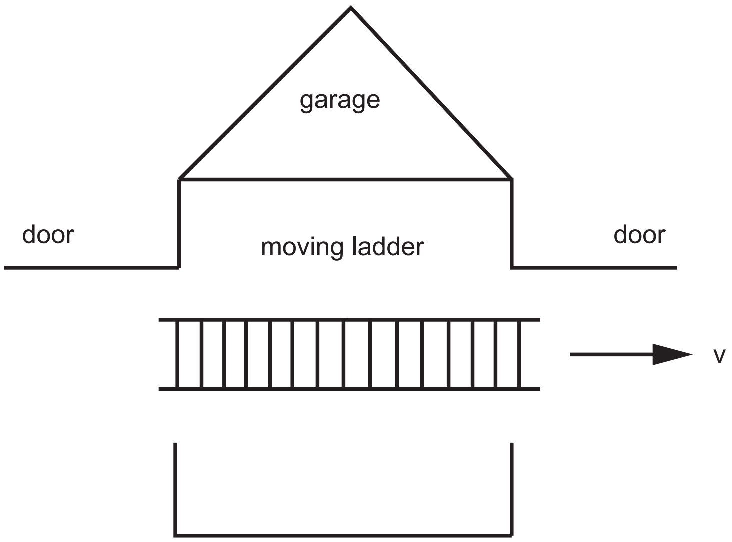
Consider a ladder, a garage and observer.

All at rest relative to each other



When the ladder is at rest with respect to the garage a stationary observer measures the length of the ladder and the width of the garage. The length of the ladder is greater than the width of the garage so the ladder cannot fit inside the garage.

Ladder moving relative to garage and observer

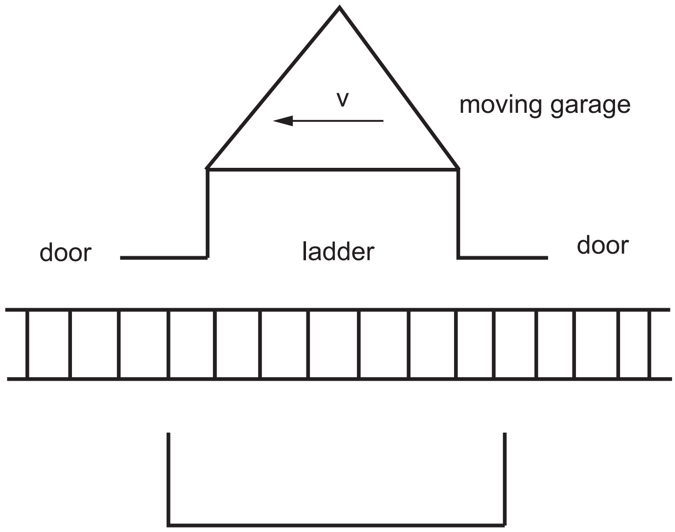


The ladder is moved through the garage at a constant speed close to the speed of light.

Because of length contraction, a stationary observer sees the ladder becoming shorter and the ladder fits inside the garage.

The observer and the garage are in the same frame of reference of the garage. This means the observer is looking from the "point of view" of the garage.

Ladder and observer moving relative to garage



The ladder and observer are moved through the garage at a constant speed close to the speed of light. Because the observer is moving with the ladder, the observer sees the garage moving in the opposite direction.

The observer sees the garage as being length contracted. This length contraction of the garage means that the ladder will not fit inside the garage.

The length of the ladder does not change for the observer who is stationary relative to the ladder.

The observer and the ladder are in the same frame of reference. This means the observer is looking from the "point of view" of the ladder.

Does the ladder fit inside the garage?

The answer depends on the motion of the observer. Both explanations are correct. There is no one absolute solution. Because the two solutions do not agree but are both correct, it is called a paradox.

The example highlights the need for you to be clear on the motion of the observer.

Experimental confirmation of length contraction

When cosmic rays arrive in at the Earth's atmosphere from space they collide with atom s. One of the results of these collisions is the production of a type of particle called a muon.

Muons are unstable short lived elementary particles with a mass about 200 times that of an electron.

When a cosmic ray collides with an atom at a height of 60 km above the Earth to produce a muon, the muon travels with a speed of about 99.9% the speed of light. Muons produced in laboratory conditions are found to have a mean lifetime of about 2 micro seconds before they decay. Even at this velocity a muon would only travel about 0.5 km before it decays so virtually no muons should be detected at sea level. However we find substantial numbers of muons *are* detected at sea level.

We can explain this if we try to calculate the length contraction for a muon travelling at this velocity.

Distance in our frame of reference, l = 60 000 m.

Velocity of muon = 0.999c m s^{-1}.

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$l' = 60000\sqrt{1 - \left(\frac{0.999c}{c}\right)^2}$$

$$l' = 60000 \times 0.045$$

$$l' = 2700 \text{ m}$$

So the length contraction for the muons is 2700 m.

In the frame of reference of the muons they are travelling 2.7 km so the time elapsed in their frame of reference is $^{2700}/_{0.999c}$= 9.0 micro seconds.

This means that while most of the muons will have decayed a substantial number will be detected at sea level and in fact the number detected agrees with the predicted amount.

Example An object that has a rest length of 100 m long is travelling at 100000 m s^{-1} .

Calculate the length contraction for this object at this velocity.

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$l' = 100\sqrt{1 - \left(\frac{100000}{3 \times 10^8}\right)^2}$$

$$l' = 99.99999444 \text{ m}$$

If we repeat he calculation for an object travelling at 2 X 10^8 m s^{-1} we can see the difference.

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$l' = 100\sqrt{1 - \left(\frac{2 \times 10^8}{3 \times 10^8}\right)^2}$$

$$l' = 74.53559925 \text{ m}$$

Notice that length contraction always makes the measured length of the moving object less than its length when it was at rest. If in a calculation, you get the moving length, l', to be larger than the rest length l, it's likely that you've reversed the values of l' and l.

6.4 Summary

Summary

You should now be able to:

- describe what is meant by a frame of reference;
- understand that during the 19th century experimental evidence and theoretical consideration gave rise to a challenge to Newton's idea of absolute space;
- understand that the constancy of the speed of light led Einstein to postulate that space and time for a moving object are changed relative to a stationary observer;
- carry out calculations for time dilation and length contraction.

6.5 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

Links

- There is a lot of detail on this site if you have time to explore it:
http://www.phys.unsw.edu.au/einsteinlight/
- There is a lot of detail on this site if you have time to explore it:
http://www.phys.unsw.edu.au/einsteinlight/
- This animation shows how Jill's **time** appears to Jack. A round-trip of the light beam takes 10 seconds measured with a clock in the same frame:
http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/lightclock.swf
- A tutorial that shows how relativistic length contraction must follow from the existence of time dilation:
http://www.cabrillo.edu/~jmccullough/Applets/Flash/Modern%20Physics%20and %20Relativity/LengthContract.swf
- This is an excellent simulation summarising the effect of time dilation and length contraction:
http://www.onestick.com/relativity/

6.6 Assessment

End of topic 6 test Go online

The following test contains questions covering the work from this topic.

The following data should be used when required:
The speed of light in a vacuum c is 3.0×10^8 m s^{-1}

Q1: A spaceship is travelling at a constant speed of 10 % of the speed of light.

Calculate the time dilation in seconds for a stationary observer if an observer on the spacecraft's stopwatch records a time of 191 seconds.

. . . .

Q2: A traveller on a spacecraft measures the time elapsed during a journey as 86400.0 s. Calculate the velocity of the spacecraft in m s^{-1} if the stopwatch of a stationary observer gives a time of 86400.5 s.

. . . .

Q3: A spacecraft is travelling at a constant speed of 40 % of the speed of light. Calculate the length contraction in metres for a stationary observer if an observer on the spacecraft measures the length of the spacecraft to be 192 metres.

Topic 7

The expanding universe

Contents

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7.3 Applications of the Doppler Effect 170
7.4 Sound 171
7.5 Light and other electromagnetic radiation 171
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7.7 Assessment 175

Learning objective

By the end of this topic you should be able to:

- describe the Doppler effect on sound and light waves;
- carry out calculations on the Doppler effect for a moving source of sound;
- state that the light from objects moving away from us is shifted to longer (more red) wavelengths;
- state that the red shift of a galaxy is the change in wavelength divided by the emitted wavelength;
- state that for slowly moving galaxies, red shift is the ratio of the velocity of the galaxy to the velocity of light.

In this topic we are going to examine the Doppler effect. This is the effect you notice in the note a police siren makes when it moves towards you and then away from you. There is a change in pitch of the note. Why should this happen? Does this happen for all waves?

7.1 The Doppler effect

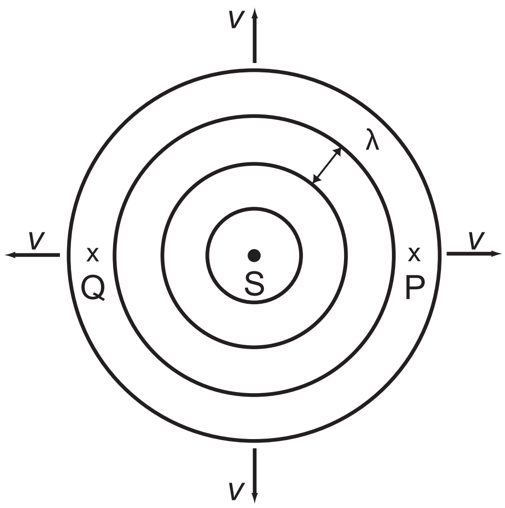
Have you ever noticed the change of note when an ambulance with a siren comes towards and goes past you, or the way that a car engine sounds different once the car has gone past you? This change in the sound waves that you hear is called the Doppler effect. The frequency of the sound waves being emitted by the siren or the engine is different to the frequency of the waves that you hear because of the relative motion of the source of the waves (the siren, the engine or whatever) and the observer or listener.

In this section we will be investigating the Doppler effect to find out how this change in frequency occurs.

7.2 The Doppler effect with moving source

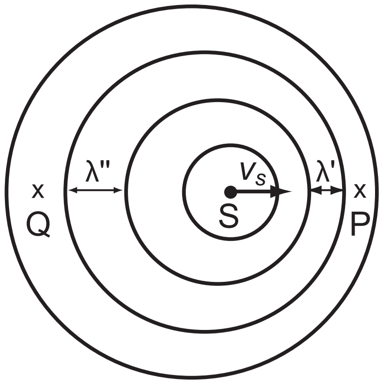
Consider a source S producing sound waves with wavelength λ and frequency f. Figure 7.1 shows the waves being emitted in all directions. P and Q are two stationary observers listening to the waves. Both observers will hear sound at the same frequency f.

Figure 7.1: Waves produced by a stationary source



The waves travel at speed v, where $v = f\lambda$, and the source S is stationary. Now let us consider what happens if S is moving whilst emitting waves.

Figure 7.2: Waves produced by a moving source



In Figure 7.2, S is moving to the right with speed v_s. The waves produced in the direction SP are compressed into a smaller space than before, as the source is moving in the same direction as the waves. λ' is therefore smaller than the emitted wavelength λ. In the direction SQ, the wavelength appears to be greater, as the source is moving in the opposite direction to which the waves are travelling. This means that λ'' is greater than λ.

When we are dealing with sound waves, the frequency of the wave tells us its pitch. Does this observed change of wavelength mean that the frequency of the waves has shifted?

Let us look first at the situation where the source S is moving towards the observer P. In a period of time t, the number of waves emitted by S is $f_s\ t$. The first wave emitted has travelled a distance vt in this time, whilst in the same time the source has moved a distance $v_s t$ in the same direction. This means that $f_s\ t$ waves are compressed into a distance $vt - v_s t$ = $(v - v_s)t$.

The distance between the waves is equal to the wavelength λ' observed by an observer at P, hence

$$\lambda' = \frac{(v - v_s)\,t}{f_s\ t}$$
$$\therefore \lambda' = \frac{(v - v_s)}{f_s}$$

Since the waves are travelling at speed v, then the observed frequency ***f***' is given by

$$\lambda' = \frac{v}{f'}$$

Substituting for λ',

$$\frac{v}{f'} = \frac{(v - v_s)}{f_s}$$
$$\therefore \frac{f'}{v} = \frac{f_s}{(v - v_s)}$$
$$\therefore f' = f_s \frac{v}{(v - v_s)}$$

(7.1)

So the frequency of the waves heard at P depends on the speed v_s of the source. This frequency shift, called the **Doppler effect**, means that the observed frequency is higher than the actual frequency of the emitted waves when the source is moving towards the observer. Make sure you understand why Equation 7.1 implies that the observed frequency *f*' is greater than the emitted frequency.

We can now look at the situation when the source S is moving away from the observer at Q. As before, the source emits $f_s\, t$ waves in time t, the first of which travels a distance vt in that period of time. The source S moves a distance $v_s t$ in the opposite direction, so these waves are spread out over a distance vt+ $v_s t$ = (v $+ v_s$)t. In this case, the distance between the waves is the observed wavelength λ".

$$\lambda'' = \frac{(v+v_s)t}{f_s\; t}$$
$$\therefore \lambda'' = \frac{(v+v_s)}{f_s}$$

Again we can substitute for λ" = v/f" in this equation

$$\frac{v}{f''} = \frac{(v+v_s)}{f_s}$$
$$\therefore \frac{f''}{v} = \frac{f_s}{(v+v_s)}$$
$$\therefore f'' = f_s \frac{v}{(v+v_s)}$$

(7.2)

In this case, the motion of the source means that the observed frequency f" is less than the emitted frequency.

The observed change in frequency is normally measured using a datalogger and a microphone. The datalogger is set to measure frequency.

If the true frequency of the source is known when the observed frequency of the moving source is measured the datalogger can calculate the speed of the moving source.

Rather than remembering and using two separate Doppler equations with f' and f" the normal way that the Doppler equation is written is:

$$f_0 = f_s \left(\frac{v}{v \pm v_s} \right)$$

(7.3)

Where:

f_0 = observed frequency of sound

f_s = frequency of sound produced by source

v = velocity of sound, usually in air

v_s = velocity of moving source

$\pm$ you need to learn and remember the following

+ is used when source is moving away from observer

- is used when source is moving towards observer

Notice that the above equations **cannot** be used for a moving observer detecting sound or when considering the Doppler effect on light waves.

Doppler effect Go online

Suppose an ambulance is travelling at 15 m s^{-1}, and the siren is emitting a note at 400 Hz. If the speed of sound is 340 m s^{-1}, what is the frequency of the note heard by the observer

1. as the ambulance approaches him?
2. after the ambulance has passed him?

Quiz: Doppler effect Go online

Useful data:

speed of sound in air	340 m s^{-1}

Q1: The rise in pitch heard when a source of sound waves approaches a stationary listener is due to a shift in

a) speed.
b) frequency.
c) amplitude.
d) phase.
e) coherence.

. . . .

Q2: Granny toots her car horn to say goodbye as she drives off down the road at 10 m s^{-1}.

What is the frequency of the note heard by her grandchildren as she drives away, if the emitted sound waves have frequency 510 Hz?

a) 324 Hz
b) 337 Hz
c) 495 Hz
d) 510 Hz
e) 526 Hz

. . . .

Q3: A man is blowing a whistle, producing a note of frequency 480 Hz. The note is heard by a woman standing 50 m away. The observed frequency heard by the woman is

a) higher when the man is walking towards her.
b) unchanged when the man is walking away from her.
c) higher when she is walking away from the man.
d) lower when she stands 20 m from the man.
e) higher when she stands 20 m from the man.

. . . .

Q4: A car is being driven towards a stationary observer, with the horn sounding. The observed frequency heard by the observer is 480 Hz. If the actual frequency of the car horn is 465 Hz, how fast is the car travelling?

a) 3.20 m s^{-1}
b) 10.6 m s^{-1}
c) 15.0 m s^{-1}
d) 22.7 m s^{-1}
e) 320 m s^{-1}

7.3 Applications of the Doppler Effect

The Doppler effect can be used to:

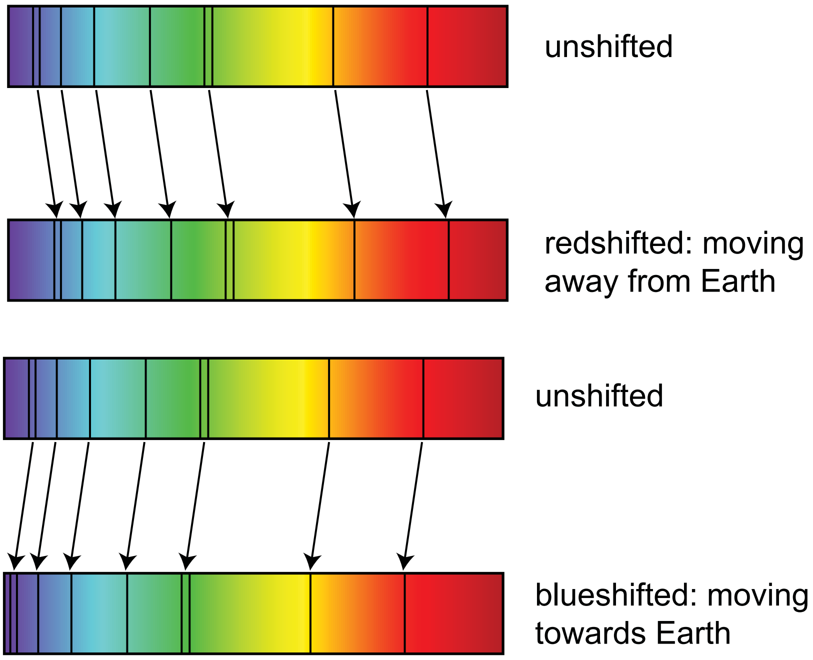
- measure the speed at which distant **galaxies** are moving relative to the Earth
- measure the speed of blood flow using ultrasound waves.

7.4 Sound

Ultrasound waves (frequencies in the MHz range, beyond the range of human hearing) are used to measure the speed of blood flow by making use of the Doppler effect. In a similar way to the radar speed gun, the flow meter emits an ultrasonic signal which is reflected by red blood cells. The reflected signal is measured by the flow meter and its frequency is compared to that of the original signal. In narrowed arteries, the blood is forced to flow faster, so the Doppler shift will be greater in such regions than in normal arteries.

7.5 Light and other electromagnetic radiation

In astronomy, the Doppler effect is used to measure the speed at which distant galaxies are moving relative to the Earth. The line spectrum of the galaxy as observed on Earth is compared to that of the appropriate elements taken in the laboratory. The lines in the galaxy's spectrum are found to be different to the laboratory spectrum, with all the wavelengths shifted in one direction. The shift in wavelength tells us how fast the galaxy is moving, and whether it is moving towards or away from Earth.



For a galaxy moving away from the Earth, lines in the galaxy's spectrum appear at a longer wavelength than the corresponding line in the laboratory spectrum.

This means that a line in the visible part of the spectrum is Doppler-shifted towards the red end of the spectrum. The line is said to be **redshifted**, and the fact that a spectrum is redshifted tells us that the galaxy is moving away from the Earth.

Similarly, a galaxy moving towards the Earth would have its spectrum **blueshifted**.

The redshift of a galaxy is the change in wavelength divided by the emitted wavelength.

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}}$$

(7.4)

Where:

z = redshift, unusual as it has no unit, it is just a number

$\lambda_{observed}$ = wavelength observed on Earth

λ_{rest} = wavelength observed when source is not moving relative to Earth

If z is positive then $\lambda_{\text{o}} > \lambda_{\text{rest}}$ so the wavelength has been redshifted and the galaxy must be moving away from us.

If z is negative then $\lambda_{\text{o}} < \lambda_{\text{rest}}$ so the wavelength has been blueshifted and the galaxy must be moving towards to us.

For slow moving galaxies, redshift (z) is the ratio of the velocity of the galaxy (v) to the velocity of light (c).

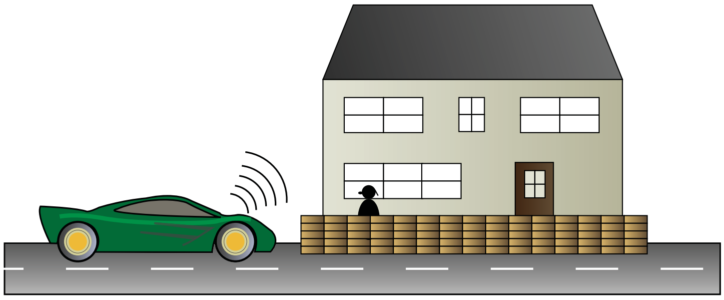
$$z = \frac{v}{c}$$

(7.5)

Another application of the Doppler effect for electromagnetic radiation is in Doppler radar. In this case microwave radiation with frequency of the order of 10^9 Hz is used. A signal is emitted from a radar gun towards a car travelling towards the gun. Since the car is moving towards the stationary source, the waves are Doppler shifted as they arrive at the car, and the reflected signal is at the new frequency. The reflected signal is detected by a receiver attached to the gun, and since it comes from a source moving towards a stationary detector, it suffers a second Doppler shift. The shift in frequency of the detected signal tells the gun operator the speed of the car.

Examples

1. The driver of a sports car sounds its horn as she approaches a building where a person is standing by the roadside as shown.



The speed of the car is 25.0 m s^{-1} and the frequency of the sound emitted by the horn is 1250 Hz. Assume the speed of sound in air is 340 m s^{-1}.

1. Explain in terms of wavefronts why the sound heard by the person does not have a frequency of 1250 Hz.
2. Calculate the frequency of the sound from the car heard by the person.

1. As the source of waves is moving towards the person, each successive wavefront is slightly closer than if the source had been stationary, therefore the frequency observed by the person is higher. The person detects more than 1250 wavefronts every second.
2.

$$f_o = f_s\left(\frac{v}{v \pm v_s}\right)$$

$$f_o = 1250x\left(\frac{340}{(340-25)}\right)$$

$$f_o = 1349Hz$$

The frequency is 1349 Hz

. . . .

2.

The spectrum of light from most galaxies contains lines corresponding to helium gas. Figure (a) shows the helium spectrum from the Sun. Figure (b) shows the helium spectrum from a distant galaxy.

Figure a)

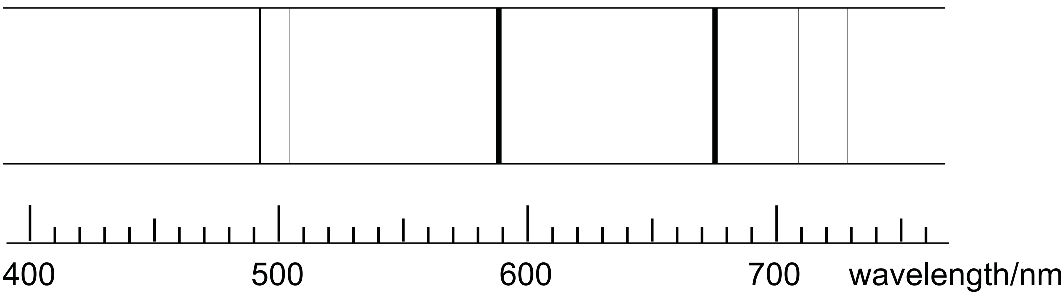
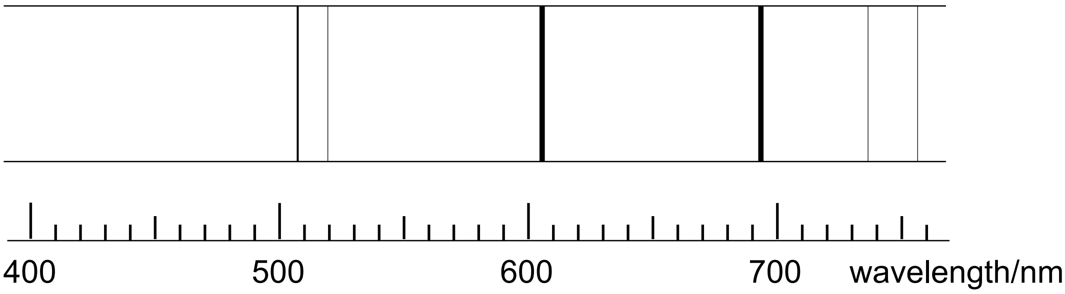


Figure b)



By comparing these spectra, what conclusion can be made about the distant galaxy? Justify your answer.

It is moving away because the wavelength is increased OR frequency decreased

7.6 Summary

Summary

You should now be able to:

- describe the Doppler effect on sound and light waves;
- carry out calculations on the Doppler effect for a moving source;
- state that the light from objects moving away from us is shifted to longer (more red) wavelengths;
- state that the redshift of a galaxy is the change in wavelength divided by the emitted wavelength;
- state that for slowly moving galaxies, red shift is the ratio of the velocity of the galaxy to the velocity of light.

7.7 Assessment

End of topic 7 test Go online

The following test contains questions covering the work from this topic.

The following data should be used when required:

speed of light in a vacuum c	*3.00 x 10 ^{8} m s^{-1}*
speed of sound	*340 m s $^{-1}$*
acceleration due to gravity g	*9.8 m s^{-2}*

Q5: The Andromeda Galaxy has a velocity of 300 km s $^{-1}$ relative to the earth.

1. Calculate the value of its redshift.
2. If light with a rest wavelength of 525 nm is omitted from this galaxy what will be its observed wavelength in nm at the Earth?

. . . .

Q6: The Sombrero galaxy has a red shift of 0.003416.

1. Calculate this galaxy's velocity in m s $^{-1}$ relative to the Earth.
2. If light with a rest wavelength of 580 nm is omitted from this galaxy what will be its observed wavelength in nm at the Earth?

. . . .

Q7: A car is being driven towards a stationary pedestrian at 12.5 m s $^{-1}$. The car driver sounds the horn, which produces a note at frequency 480 Hz.

Calculate the frequency in Hz of the note heard by the pedestrian.

. . . .

Q8: The siren of a fire engine is Doppler-shifted from 507 Hz to 482 Hz as it drives away at constant speed from a stationary observer.

Calculate the speed of the fire engine, in m s $^{-1}$.

. . . .

Q9: A car travelling at constant speed passes by a stationary pedestrian as the car driver sounds the horn. As the car approaches the pedestrian she hears the horn sounding at 539 Hz. Once the car has driven past her, she hears the horn sounding at 461 Hz.

1. Calculate the speed of the car, in m s $^{-1}$.
2. Calculate the frequency in Hz of the sound waves emitted by the car horn.

Topic 8

Hubble's Law

Contents

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8.2 Hubble's law . . . 179
8.3 Summary . . . 183
8.4 Extended information . . . 183
8.5 Assessment . . . 184

Learning objective

By the end of this topic you should be able to:

- state that Hubble's Law shows the relationship between the recession velocity of a galaxy and its distance from us;
- state that Hubble's Law leads to an estimate of the age of the universe;
- state that the mass of a galaxy can be estimated from the orbital speed of the stars within it;
- state that this estimated mass is greater than the mass that can be observed;
- state that this estimated mass gives rise to concept of dark mass;
- state that the rate of expansion of the universe is accelerating with time;
- state that it has been proposed that this acceleration is due to dark energy.

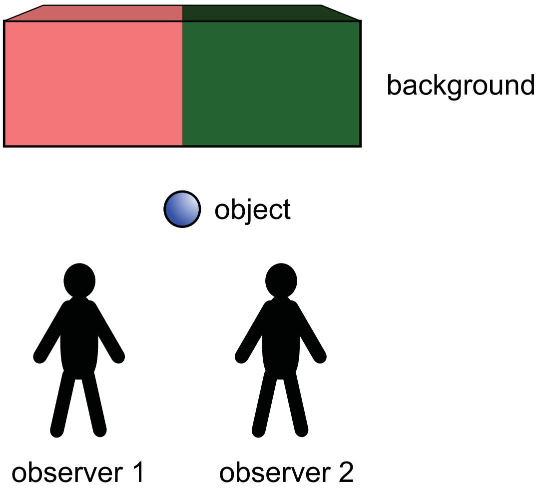
In this topic we will see how this effect is used to measure the velocity of galaxies and the startling conclusion reached by Hubble when he collected data on the velocities of galaxies and their distance from our own. Finally we will see how Hubble's Law enables us to estimate the age of the Universe.

8.1 Measuring large distances in space

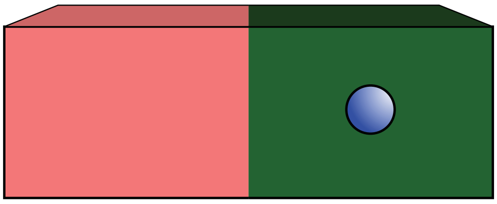
In practice it is difficult to measure the distance to distant objects.

The principle of **parallax** can be used to measure the distance of objects that are relatively close to the Earth. When an object is viewed against a distant background by two observers at different positions they will see a different image.

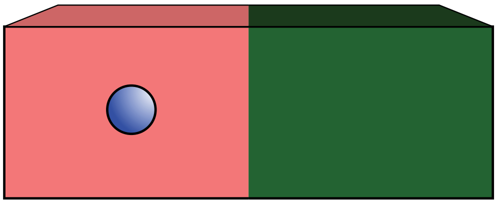
Consider two observers looking at a blue ball against a two coloured background as shown.



Observer 1 will see the following image.



But observer 2 will see the following image.



This effect is called parallax.

The distance of the object affects the amount of parallax. The closer the object is to the observer the greater the parallax. If the angle is measured and the distance between the two observers is known the distance to the object can be calculated using trigonometry.

Parallax is used to measure distances to stellar objects by measuring the position of stars at different times of the year, that is at different points of the earth's orbit around the sun.

It has to be emphasised that the parallax effect is very small and it is only with very accurate instruments that the very large distances can be measured. Parallax is now normally used to calibrate other distance measuring devices.

Another method of measuring very large distances is to use what is known as a **standard candle**.

There are distant stellar objects that we know have a certain, fixed brightness. If we measure their apparent brightness from the earth we can calculate how far away they are by comparing their apparent brightness to their actual brightness. This is possible because if we double the distance from an object then its apparent brightness becomes a quarter of what it was before.

Because the distances are so large astronomers sometimes use different units to describe these distances.

The light year is the distance light travels in one year.

This can be calculated as shown below

$s = vt$
$s = 3 \times 10^8 \times (60 \times 60 \times 24 \times 365)$ $s = 9460800000000000$ m

The nearest star to the Earth is 4.2 light years away.

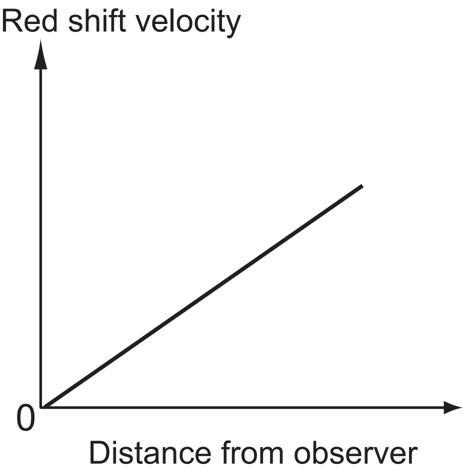
The **parsec** is also used by astronomers.

Parsecs were first used when measuring distances using parallax (parsec stands for parallax arc second). The parsec is equivalent to 3.26 light years. (30842208000000000 m).

8.2 Hubble's law

During the 1920s Edwin Hubble measured the Doppler shift of many observable galaxies and found that they were all red-shifted. This meant that all of the galaxies were moving away from us. He also measured their rate of divergence and found that the further away they were from us the faster that they were receding. This is due to the expansion of space and is considered to be evidence for the Big Bang as the origin of the universe.

If a graph of red-shift velocity against distance is plotted from distance from the observer is plotted the following graph is obtained.



This gives rise to **Hubble's law**: a galaxy's velocity (sometimes called the *recessional velocity*, the velocity at which the galaxy is moving away from Earth) is proportional to the distance from the observer. Mathematically this gives rise to the expression

$$v = H_o d$$

where H_o is the Hubble constant.

(8.1)

Attempts to measure Hubble's constant have given a range of values. The valued used in this course will be $2.3 \times 10^{-18}\ s^{-1}$.

We can use Hubble's law to make an estimate of the age of the universe.

We have already seen that Hubble's law is regarded as evidence for the Big Bang theory of the beginning of the universe.

We already know that $v = d/t$

Hubble's law states that $v = H_o d$

Therefore $H_o d = d/t$

So $t = 1/H_o$

Using the current estimate for Hubble's constant we can make an estimate of the age of the universe. This gives a value based on estimates of Hubble's constant of between 1×10^{10} and 2×10^{10} years. As we shall see this is within the range of other measurements used to make an estimate of the age of the universe.

Examples

1.

A galaxy is found to be moving away from the Earth at a speed of 25 Mm s^{-1}. Calculate the distance from the Earth to the galaxy.

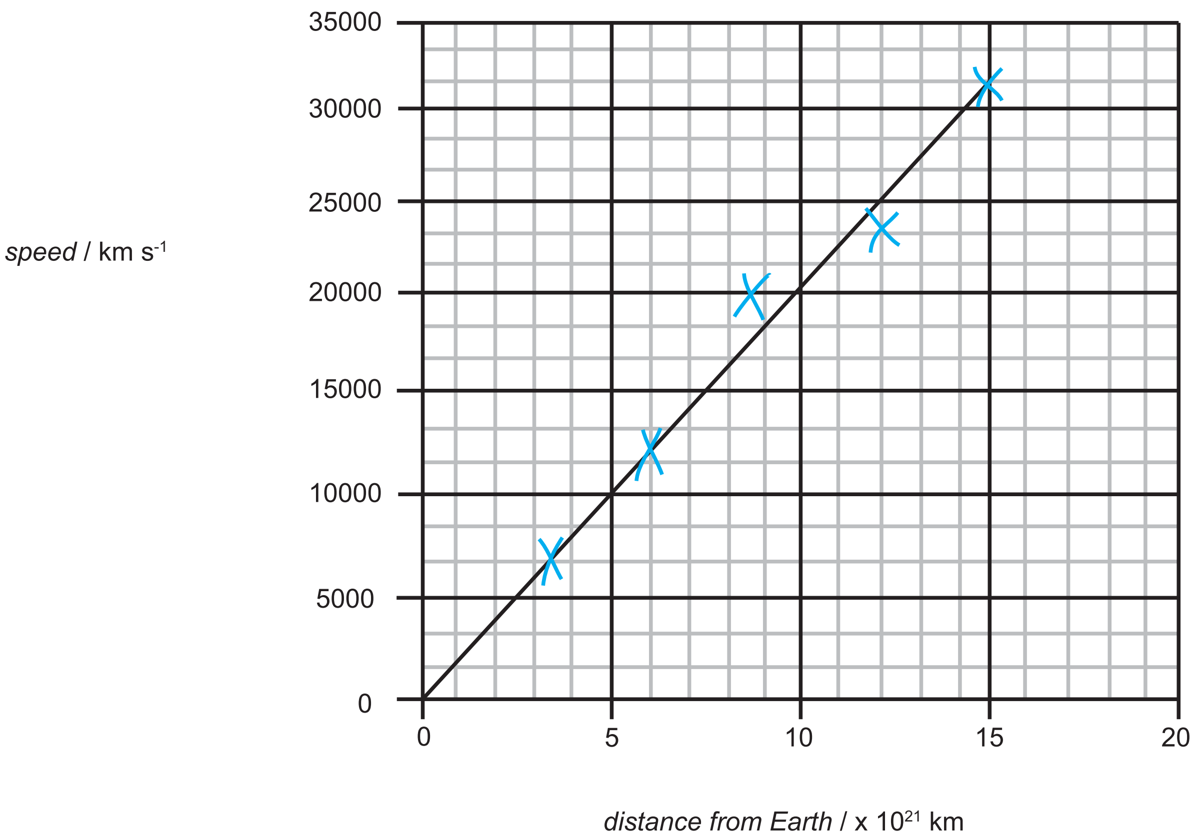
$$v = 25\ Mm\ s^{-1}$$
$$H_O = 2.3 \times 10^{-18}\ s^{-1}$$
$$d = ?$$

$$v = H_O \times d$$
$$25 \times 10^6 = 2.3 \times 10^{-18} \times d$$
$$d = 1.1 \times 10^{25}\ m$$

....

2.

The following graph shows how the recessional velocity of different galaxies varies with distance from the Earth.



Use this graph to find a value of Hubble's constant.

Since v = H_O x d then H_O will equal the gradient of this graph. (Take care with the powers of 10.)

$$gradient = \frac{y_2 - y_1}{x_2 - x_1}$$
$$gradient = \frac{31000 \times 10^3 - 0}{15 \times 10^{24} - 0}$$
$$gradient = 2.1 \times 10^{-18} s^{-1}$$
$$H_O = 2.1 \times 10^{-18} s^{-1}$$

....

3.

All galaxies are moving away from the Earth. The distance, d, a galaxy is from the Earth is related to the recessional velocity, v, of the galaxy.

The table gives the distances of four galaxies from the Earth and the recessional velocity of each galaxy.

Distance from the Earth d / 10^{23} m	*Recessional velocity v / 10^6 m s^{-1}*
5.32	1.24
8.21	1.87
10.4	2.38
18.4	4.28

Use all of the data in the table to show that the relationship between the recessional velocity, v, of a galaxy and the distance, d, of the galaxy from the Earth is

$$v = 2.3 \times 10^{-18} \times d$$

For each set of data calculate:

$$\frac{v}{d}$$

$$\frac{1.24 \times 10^6}{5.23 \times 10^{23}} = 2.33 \times 10^{-18}$$

$$\frac{1.87 \times 10^6}{8.21 \times 10^{23}} = 2.28 \times 10^{-18}$$

$$\frac{2.38 \times 10^6}{10.4 \times 10^{23}} = 2.29 \times 10^{-18}$$

$$\frac{4.28 \times 10^6}{18.4 \times 10^{23}} = 2.33 \times 10^{-18}$$

Concluding:

$$\frac{v}{d} = 2.3 \times 10^{-18} \Rightarrow v = 2.3 \times 10^{-18} \times d$$

8.3 Summary

Summary

You should now be able to:

- state that Hubble's Law shows the relationship between the recession velocity of a galaxy and its distance from us;
- state that Hubble's Law leads to an estimate of the age of the Universe;
- carry out calculations using Hubble's law;
- describe the evidence for the expansion of the universe and how this relates to the concepts of dark matter and dark energy.

8.4 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

Links

- This site provides an explanation of Hubble's law:
  http://astrosun2.astro.cornell.edu/academics/courses//astro201/hubbles_law.htm
- A good site for Hubble's law and some interesting further study throught the links provided on the site:
  http://hyperphysics.phy-astr.gsu.edu/hbase/astro/hubble.html
- This is a simulation showing the balloon analogy of the universe:
  http://www.astro.ucla.edu/~wright/balloon0.html
- This website replaces the balloon analogy with an elastic band analogy which allows exemplification of other points:
  https://www.e-education.psu.edu/astro801/content/l10_p4.html

8.5 Assessment

End of topic 8 test Go online

The following test contains questions covering the work from this topic.

The following data should be used when required:

speed of light in a vacuum c	*3.00 x 10 ^{8} m s^{-1}*
speed of sound	*340 m s $^{-1}$*
acceleration due to gravity g	*9.8 m s^{-2}*

The end of topic test is available online. If however you do not have access to the web, you may try the following questions.

Q1:

The galaxy, UGC 858 has a velocity relative to the Earth of 2.375 x 10^{6} m s^{-1} and is at a distance of 1.11 x 10^{24} m.

Calculate the value that observations of this galaxy give for Hubble's constant.

. . . .

Q2: The Cartwheel galaxy has a velocity relative to the Earth of 9050 km s^{-1} and is at a distance of 500 million light years.

Calculate the value that observations of this galaxy give for Hubble's constant in km s $^{-1}$ / million light years.

Topic 9

Expansion of the universe

Contents

9.1 Evidence for the expanding universe 186
9.2 Mass of the universe 187
9.3 Dark matter 187
9.4 Dark energy 188
9.5 Summary 189
9.6 Extended information 189
9.7 Assessment 190

Learning objective

By the end of this topic you should be able to:

- discuss evidence for the expanding universe;
- state that the mass of a galaxy can be estimated from the orbital speed of the stars within it;
- state that this estimated mass is greater than the mass that can be observed;
- state that this estimated mass gives rise to the concept of dark mass;
- state that the rate of expansion of the universe is accelerating with time;
- state that it has been proposed that this acceleration is due to dark energy.

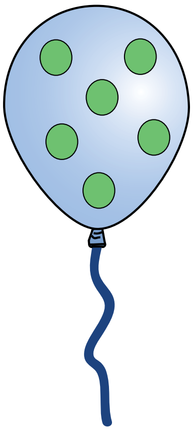
In this topic we will consider the evidence for the expansion of the universe. A method of estimating the mass of a galaxy will then be discussed. Finally we will consider whether the universe will continue to expand or if it will eventually contract.

9.1 Evidence for the expanding universe

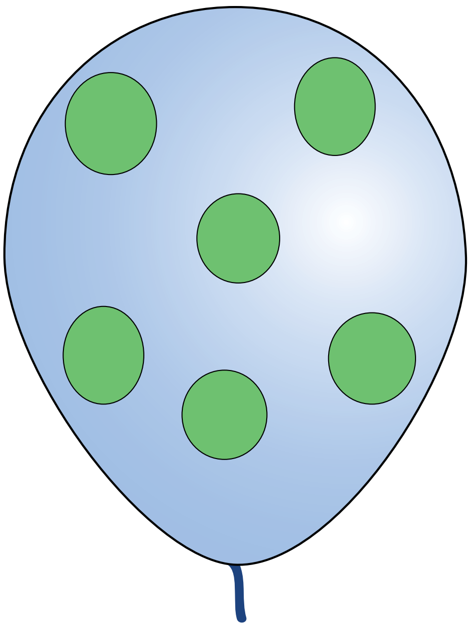
We have already seen in the previous section that all the other galaxies in the universe are moving away from us and that the greater the distance from us the faster they are moving away.

This is a clear indication that the universe is expanding. Space itself is expanding.

One way of trying to imagine what is happening is to think of a balloon with spots drawn on its surface.



If we now allow more air into the balloon we can see that the spots are now further apart.



The balloon has expanded so the spots are now further apart. In the universe the galaxies (the spots) are getting further apart so we conclude the universe (the balloon) is expanding.

Remember, the redshift of light coming from other galaxies is evidence that they are moving away from us.

As with all analogies, this balloon model shows some points correctly but it has its limitations. This balloon model of the expanding universe correctly shows the increase in distance between the galaxies. However, it shows the size of the spots (the galaxies) also increasing and this is not true. The size of individual galaxies is not increasing, they remain the same size.

The force that is causing the expansion of the universe to slow down is the force of gravity. All of the particles in the universe exert a force on each other. For small particles and groups of particles this is very small and can often be ignored but for large objects such as planets and stars and in particular collections of stars - galaxies - this force is very large. This force of attraction is causing the expansion of the universe to slow down.

The question remains as to whether the gravitational attraction in the universe will be large enough to overcome the expansion or not. This will ultimately depend on the total mass of all the particles in the universe. If this is too low the universe will go on expanding forever. If there is enough mass then at some point the universe will begin to contract.

9.2 Mass of the universe

The mass of our galaxy is estimated by measuring orbital speeds of the sun and other stars around the galactic centre. This is a reliable method of measuring the mass of the galaxy as the orbital speed of a star is almost entirely dependent of the gravitational pull due to the matter inside the galaxy. The larger the mass, the greater the pull and therefore the greater orbital velocity of the star.

A recent estimate of the total mass of our galaxy gives a value of 6 to 10 X 10^{11} times the mass of the sun. (A mass of approximately 10^{42} kg)

The estimate for the mass of our galaxy (and others) gives a value much higher than the amount of mass that is visible. This has led to the idea that there is a great deal of matter in the universe that does not emit electromagnetic radiation such as visible light. This is referred to as **dark matter**.

9.3 Dark matter

In order to explain why the total mass of a galaxy can be greater than the total mass that is observed physicist have put forward the idea of dark matter.

Normal visible matter makes up only about 1/6th of the predicted mass required to explain the orbital speeds of stars and planets. The other 5/6th of the matter is predicted to come from dark matter.

The total of the mass of each galaxy gives the total mass of the universe. The mass of the universe therefore mainly consists of dark matter.

This missing mass cannot be seen or detected as any form of electromagnetic radiation.

We know that:

- it is not matter (mass) in the form of planets or stars;
- it is not in the dark clouds of normal matter which are made up of particles;
- it is not antimatter because we could detect the gamma rays that would be emitted when antimatter annihilates with matter, see Topic 2;
- finally, it is not in the form of black holes as we can observe the effect black holes have on light and this does not happen frequently enough to explain the missing mass.

We do not know what dark matter is made of but the evidence from the orbital speed of stars and planets around the centre of the galaxy predicts that there must be more mass in the galaxy than we can observe.

Dark matter is an attempt to explain the motion of these stars and planets in terms of our known physics.

9.4 Dark energy

When the expansion rate of the universe was measured in 1998 it is found to be increasing with time.

The force of gravity pulls mass together therefore gravity would tend to slow down the rate at which the universe is expanding.

The increasing rate of expansion of the universe has given rise to the idea that the force of gravity is being overcome by some other source of energy. In order to explain why the rate of expansion is increasing physicists have put forward the concept of **dark energy**.

Again the "dark" refers to the fact that we cannot detect the source of the energy but the increasing rate of expansion of the universe is evidence that supports its existence.

This dark energy provides the energy for the increasing rate of expansion of the universe.

Was Einstein correct?

When Einstein devised the General Theory of Relativity he included a 'cosmological constant', L, in his equations to explain observations of the universe. When Hubble carried out his work analysing the measurement of galactic velocity and distance, he thought that Einstein had made a mistake and abandoned the idea of the cosmological constant.

However recent measurements that suggest that the universe is expanding have caused astronomers to think that Einstein may have been correct in the first instance and that a cosmological constant should be reintroduced. This is yet another point that has not been confirmed.

9.5 Summary

Summary

You should now be able to:

- describe the evidence for the expansion of the universe;
- discuss the significance of dark matter in the estimation of the total mass of a galaxy and the universe;
- discuss the importance of dark energy in the explanation of the increase in rate of expansion of the universe.

9.6 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date.
They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

Links

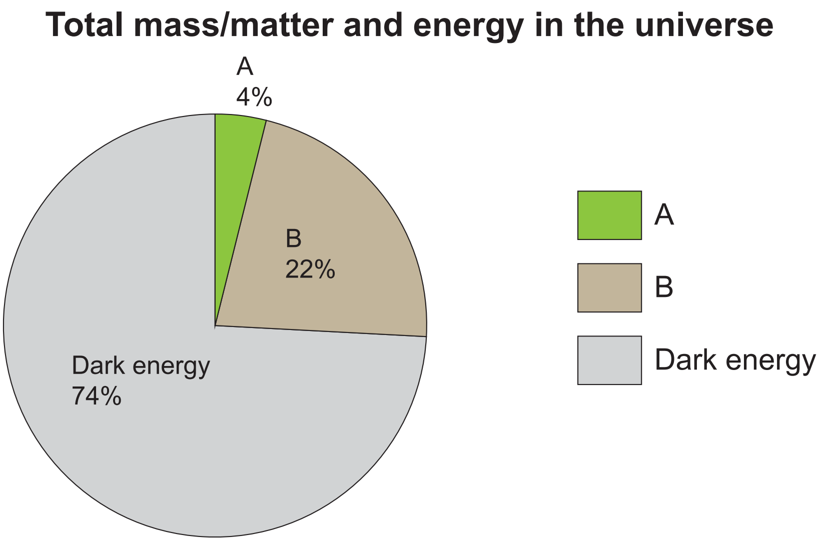
- This is a simulation showing the balloon analogy of the universe:
  http://www.astro.ucla.edu/~wright/balloon0.html
- This website replaces the balloon analogy with an elastic band analogy which allows exemplification of other points:
  https://www.e-education.psu.edu/astro801/content/l10_p4.html

9.7 Assessment

End of topic 9 test Go online

Q1:

Shown below is a pie chart of the total mass/matter and energy in the universe. Two segments have not been labelled.



a) Name the type of mass/matter or energy that is represented by segment A.

b) Name the type of mass/matter or energy that is represented by segment B.

. . . .

Q2: Which one of the following provides evidence of the expansion of the universe?

a) Blue shift of light
b) Red shift of light
c) Dark matter
d) Dark energy

. . . .

Q3: The existence of dark matter has been suggested to explain the:

a) expansion of the universe.
b) mass of the universe.
c) rate of expansion of the universe.

. . . .

Q4: The rate of expansion of the universe is increasing. It has been proposed that this increase is due to the existence of:

a) the gravitational attraction between galaxies.
b) dark matter.
c) dark energy.

Topic 10

The Big Bang theory

Contents

10.1 Measuring temperature 192
10.2 The temperature of stellar objects 193
10.3 Evidence for the Big Bang 196
10.4 Summary 197
10.5 Extended information 197
10.6 Assessment 198

Learning objective

By the end of this section you should be able to state that :

- all stellar objects give out a wide range of wavelengths of electromagnetic radiation but that each object gives out more energy at one particular wavelength;
- the wavelength of this peak wavelength is related to the temperature of the object with hotter objects having a shorter peak wavelength than cooler objects;
- peak wavelengths allow the temperature of stellar objects to be calculated;
- hotter objects also emit more radiation per unit surface area at all wavelengths than cooler objects;
- the Universe cools down as it expands;
- the peak wavelength of cosmic microwave background allows the present temperature of the Universe to be determined and that this temperature corresponds to that predicted after the Big Bang, taking into account the subsequent expansion and cooling of the Universe;
- the expansion of the Universe can explain why the night sky is dark (Olber's paradox).

In the previous topic we looked at evidence for an expanding universe.

The conclusion that scientists have reached as a result of this is that not only is the Universe expanding but also that it came into existence about 14 billion years ago in an event called the Big Bang.

This topic will examine some of the supporting evidence for these conclusions.

10.1 Measuring temperature

All objects that are at a higher temperature than their surroundings emit infra red radiation. The greater the difference in temperature the more infra red radiation is given out.

This principle is often used to measure the temperature of very hot objects.

If we try to measure the temperature of a pottery kiln or a furnace with an ordinary laboratory thermometer it would shatter due to the high temperature. To measure the temperature of these objects we use an infrared detector placed at a distance from the surface you are trying to measure.

This is sometimes known as **remote sensing**.

One type of thermometer that uses this principle is shown below.

The thermometer has two laser diodes that give off narrow converging beams of light. The position of the thermometer is moved back and forth until the two beams meet at the surface of the object. The infra red sensor is now at the correct distance from the hot surface and the temperature of surface can be read from the display.

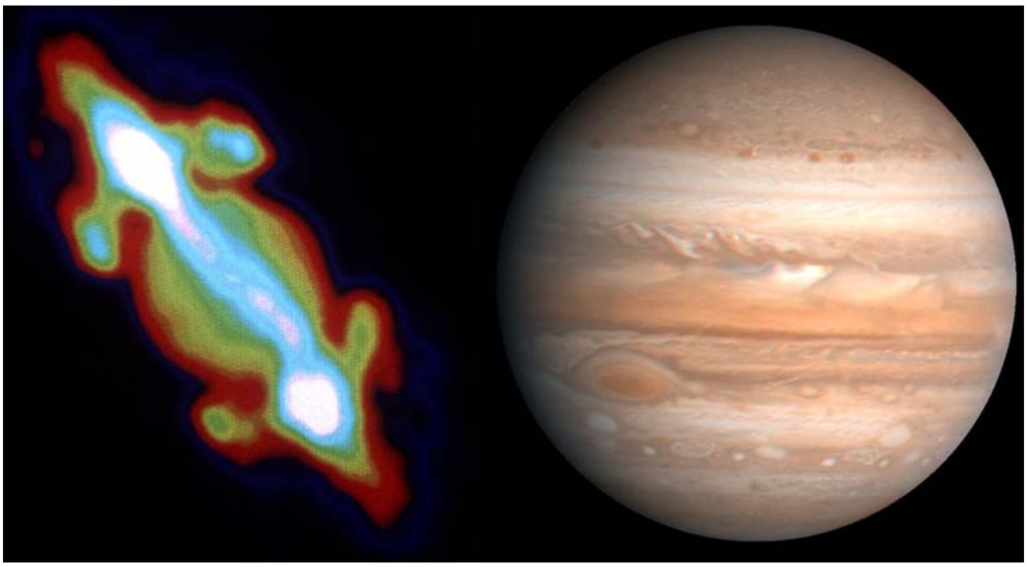
When some objects become hot they start to glow and give off light. The colour of the light given off depends on the temperature of the object. This can be used to measure the temperature of steel. When steel is being worked in a forge its colour changes with temperature. This chart shows the colour steel appears at different temperatures.

1200ºC	White
1100ºC	Light yellow
1050ºC	Yellow
980ºC	Light orange
930ºC	Orange
870ºC	Light red
810ºC	Light cherry
760ºC	Cherry
700ºC	Dark cherry
650ºC	Blood red
600ºC	Brown red

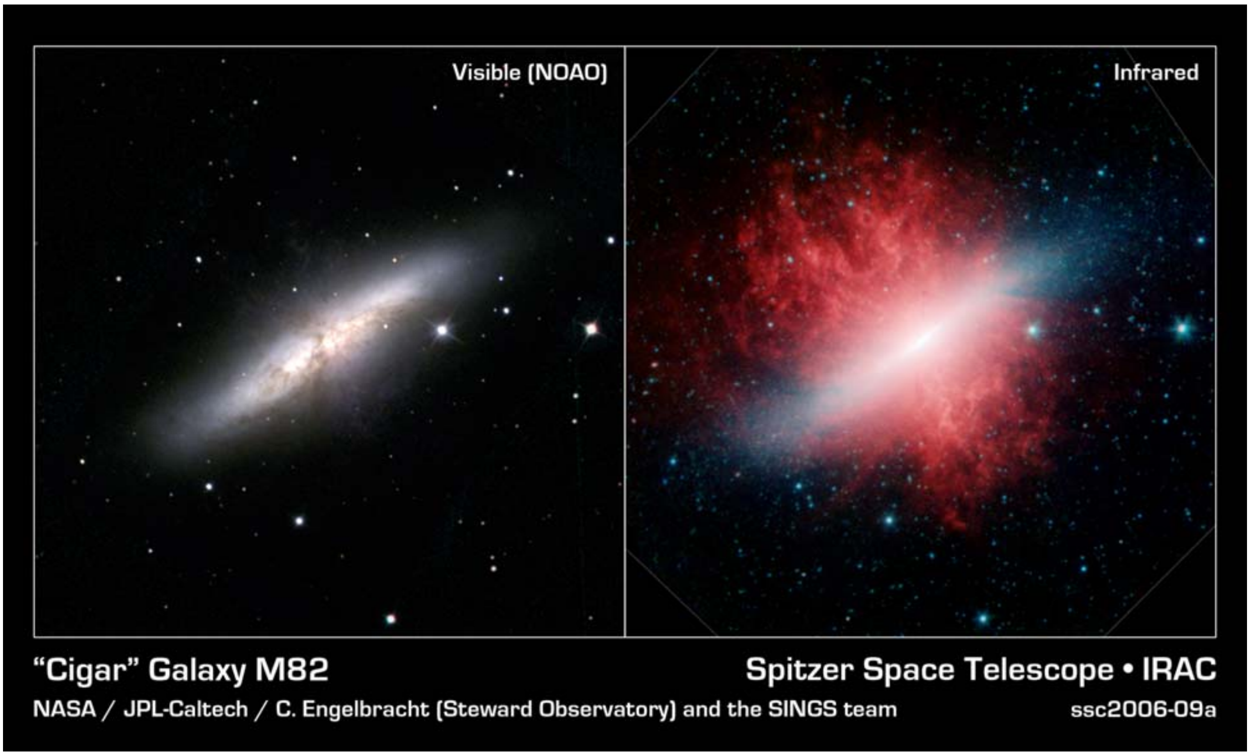
This is useful to engineers who wish to be sure that the steel is at the correct temperature before they start to forge it.

10.2 The temperature of stellar objects

Stars give out a wide range of wavelengths of electromagnetic radiation. Our own sun gives off a wide range of electromagnetic radiations including visible light, ultraviolet and infra red radiations. Some stars that only give off small quantities of visible light may give out much larger quantities of other electromagnetic radiations such as radio waves or x-rays. Astronomers will use many types of telescopes detecting different types of radiation when looking at the Universe. For example a radio telescope image and a visible light image of the same part of the sky may look very different.

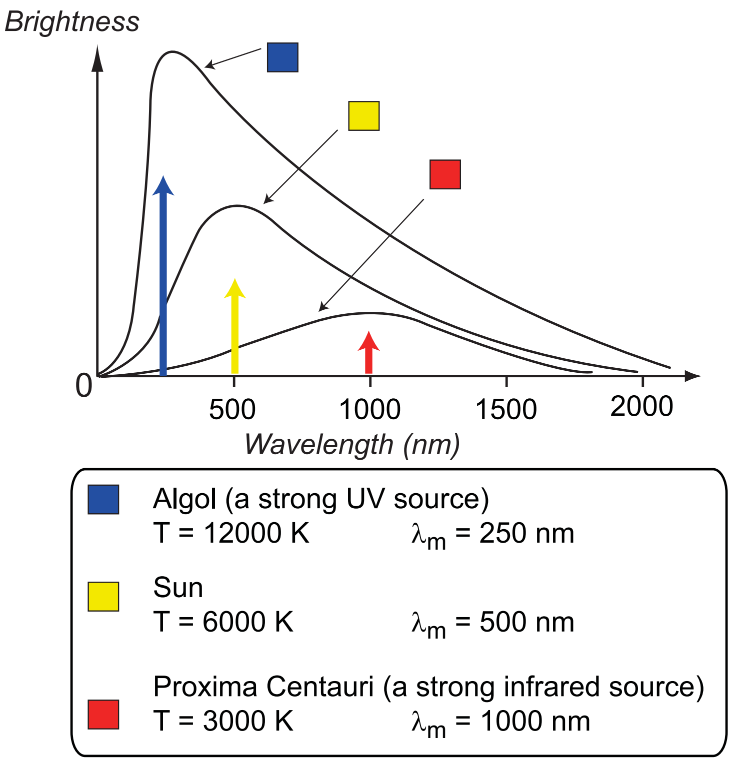


This pair of images shows the difference in a radio telescope image and a visible light image of the planet Jupiter.



These images show the same galaxy viewed in visible and in infrared radiation.

The energy that stars emit is spread over a wide range of wavelengths. However there is a wavelength at which the star emits more energy than any other. This is the peak wavelength and is a measure of the temperature of the star. The smaller the wavelength of the peak wavelength, then the higher the temperature of the object. Another way of looking at this is to say that the higher the frequency of the peak then the higher the temperature of the object.



Hot objects also emit more energy per unit area *at all wavelengths* than colder objects.

It is important to recognize the shape of this graph. The y-axis can have many different labels including irradiance, intensity, radiation per unit surface area, power density or energy. No matter which label the graph has, as the temperature of the star increases:
the peak moves to the left ie to a shorter wavelength/higher frequency,
and has a greater amplitude, height.

When trying to measure the temperature of stars astronomers measure the peak wavelength and then use this value to determine the temperature of the star. This allows astronomers to identify what type of star is being observed.

Stars are often categorised by colour and size. For example red giants and blue dwarfs. Stars at the blue end of the spectrum have shorter peak wavelengths than stars at the red end of the spectrum and therefore have a higher temperature.

The table below shows one method of classifying stars known as the Harvard classification.

Class	Temperature (K)	Colour	Notes	Examples
O	$\geq$ 33,000	blue	Most of the electromagnetic radiation given out is ultraviolet	Several stars in the Orion constellation
B	10,000 - 33,000	blue to blue white	These stars are short lived	Rigel
A	7,500 - 10,000	white	This type of star is fairly common in our part of the galaxy	Sirius (one of the brightest stars in the sky) Deneb
F	6,000 - 7,500	yellowish white	Again these star types are common in our part of the galaxy	Capella
G	5,200 - 6,000	yellow	Our own star, the sun is a class G star	The Sun, Polaris (northern pole star)
K	3,700 - 5,200	orange	May be suitable for sustaining life on solar system in their orbit	Arcturus, Aldebaran
M	$\leq$ 3,700	red	The most common of all types of star	Barnard's star

10.3 Evidence for the Big Bang

One piece of evidence for the Big Bang comes from an idea called Olber's paradox.

This poses the question that if the Universe is infinitely old why is the night sky dark and not filled with light?

The fact that the night sky is dark suggests that the Universe is not static and infinitely old but came in to existence at some definite point in the past. **If the Universe was static** and populated by an infinite number of stars, any sight line from Earth must end at the (very bright) surface of a star, so the night sky should be completely bright.

The Big Bang theory predicts that **the Universe is not static** and gives rise to the following explanation of why the night sky is dark.

- The Universe is expanding, so light from distant galaxies is red-shifted into obscurity.
- The Universe is young. Distant light hasn't even reached us yet.

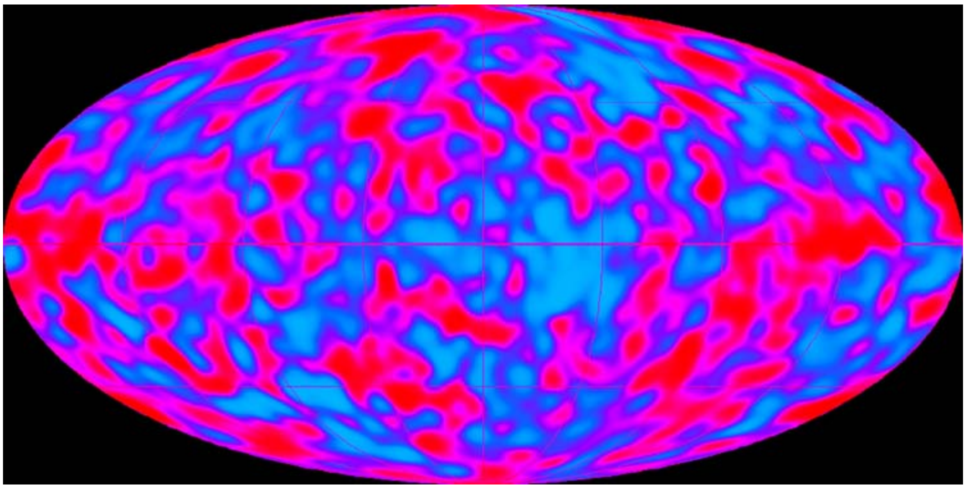
These two statements agree with the night sky appearing dark.

In the last topic we have seen that measurements show that the Universe is expanding. As it expands the Universe is cooling down.

The current temperature of the Universe can be measured by examining the peak wavelength in the cosmic microwave background radiation, CMBR.

Once the peak wavelength of the cosmic background radiation is measured the temperature is calculated in the same way as the temperature of a star.

The longer the peak wavelength the lower the temperature obtained for the background microwave radiation. This was done in part by NASA's COBE satellite which produced the following image of the cosmic microwave background radiation.



When the peak wavelength was measured it was found to give a temperature that agreed with predicted temperature when taking into account the expansion of the Universe after the Big Bang. This is further evidence in support of the Big Bang theory for the beginning of the Universe. The peak wavelength of the CMBR has been found to be about 2 mm. This predicts a temperature in the region of 2 Kelvin, -271 °C. This very low temperature is due to the fact that the Universe has been expanding and therefore cooling for a very long time.

10.4 Summary

Summary

You should now be able to explain:

- that all stellar objects give out a wide range of wavelengths of electromagnetic radiation but that each object gives out more energy at one particular wavelength.
- that the wavelength of this peak wavelength is related to the temperature of the object with hotter objects having a shorter peak wavelength than cooler objects.
- that peak wavelengths allow the temperature of stellar objects to be calculated.
- that hotter objects also emit more radiation per unit surface area at all wavelengths than cooler objects.
- that the Universe cools down as it expands.
- that the peak wavelength of cosmic microwave background allows the present temperature of the Universe to be determined and that this temperature corresponds to that predicted after the Big Bang, taking into account the subsequent expansion and cooling of the Universe.
- why the night sky is dark.

10.5 Extended information

The authors do not maintain these web links and no guarantee can be given as to their effectiveness at a particular date. They should serve as an insight to the wealth of information available online and encourage readers to explore the subject further.

Links

- This site explores different stellar types for those who want to know more detail:
  http://www.astrophysicsspectator.com/topics/observation/StellarTypes.html
- A concise explanation of big bang theory:
  http://big-bang-theory.com/
- This site includes information on how the temperature of the Universe has changed since the Big Bang. In addition it gives a timeline of the creation of all types of particles from quarks to stars and galaxies:
  http://resources.schoolscience.co.uk/PPARC/bang/bang.htm
- This site provides an explanation of the question of why the night sky is dark, Olber's paradox:
  http://www.wimp.com/skydark/

10.6 Assessment

End of topic 10 test Go online

The following test contains questions covering the work from this topic.

Q1: Two stars are being compared: Rigel which has a temperature of approximately 11000 °C and Sirius which has a temperature of approximately 25000 °C.

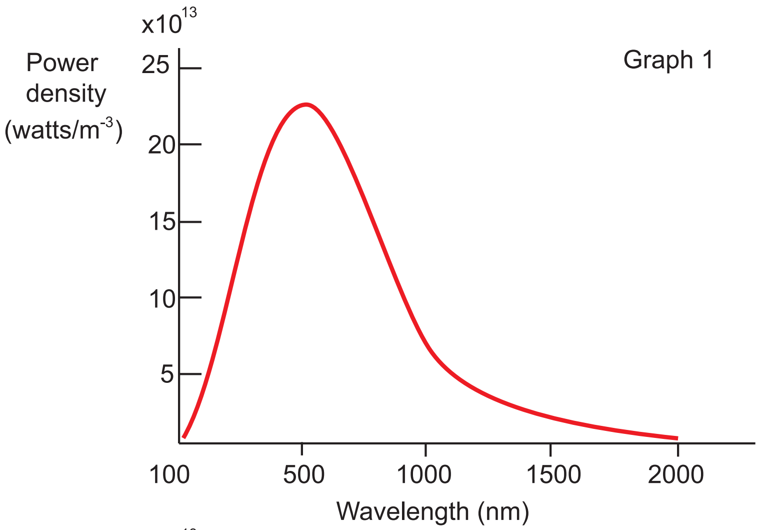
Which of these two stars gives out light with the longest peak wavelength?

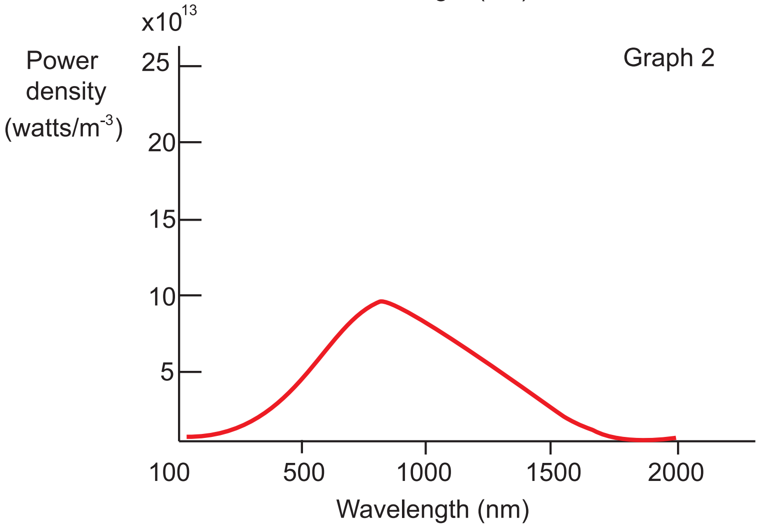
. . . .

Q2: Which group of stars has a higher temperature; red giants or blue giants?

. . . .

Q3: The following graphs show power intensity of radiation emitted by a star against the wavelength of radiation emitted.





Which graph represents the power density emitted by the hotter star?

Topic 11

End of unit tests

Contents

11.1 Open ended and skill based questions 200
11.2 Course style questions 202
11.3 End of unit assessment 207

11.1 Open ended and skill based questions

Open ended and skill based questions

Q1: A designer is developing an advert to encourage car passengers to wear seatbelts. Part of the description of the advert is as follows.

A passenger is in a car which is moving at a constant velocity of 20 m s^{-1}.
The driver is wearing a seat belt but the passenger is not.
The driver brakes suddenly. The passenger says "I was thrown towards the dashboard of the car and I was injured when I hit the dashboard."
The driver says "I seemed to stop with the car and I was not injured."

Use your knowledge of physics to comment on what is said by both the passenger and the driver.

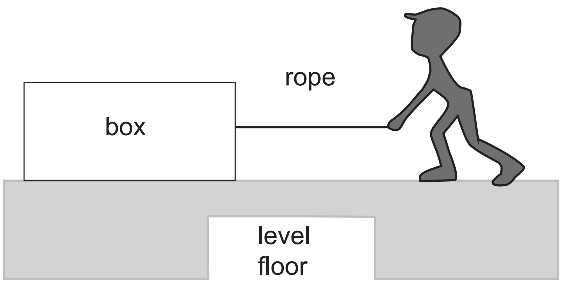
. . . .

Q2: At the end of a race a sprinter hits a crash mat and stops.

Estimate the average force applied to the sprinter by the crash mat as the sprinter is stopped. Clearly show any estimates you have made and your working for the calculation.

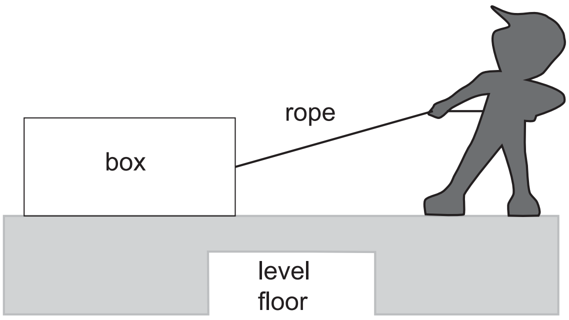
. . . .

Q3: A workman tries to move a heavy box along a level floor by applying a horizontal force with a rope as shown.



The workman is unable to move the box.

The workman then applies the same force at an angle as shown.

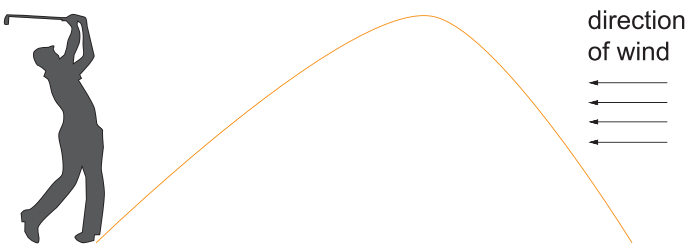


The workman is now successful in moving the box along the level floor.

Use your knowledge of physics to comment on why the workman is successful when a force is applied at an angle.

. . . .

Q4: A golfer hits a golf ball into the wind. The flight of the ball is shown.



A commentator watches the ball as it flies and gives the following description:

"The ball soars into the sky, seems to hang in the sky for a moment and then falls to the ground getting slower as it falls towards the ground."

Use your knowledge of physics to comment on what is said by the commentator.

. . . .

Q5: The following is from a report a submarine disaster:

Submarines are built to survive the high pressures experienced underwater. This submarine went below its safe operating depth and the hull groaned, until finally, deafeningly gave way to the massive pressure exerted by the water.

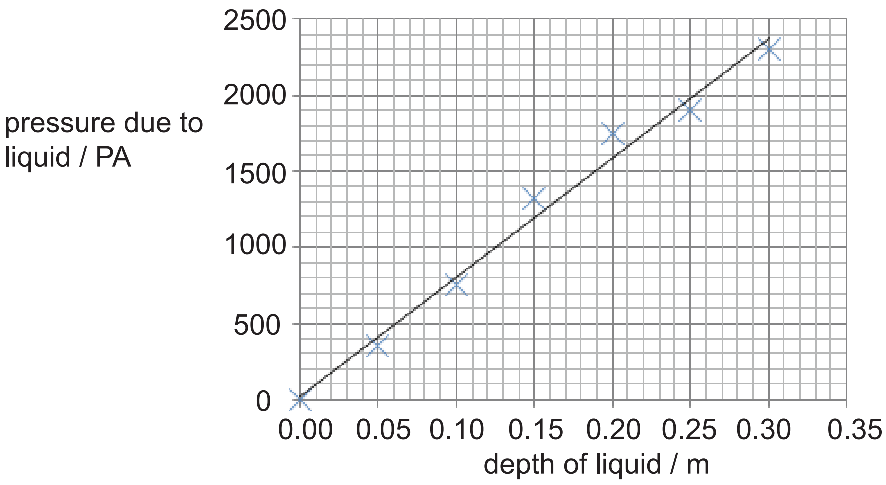
The pressure exerted by a liquid is given by the relationship

$$P = \rho g h$$

where

- P is the pressure exerted by the liquid in pascals, Pa;
- ρ is the density of the liquid in kg m^{-3};
- g is the acceleration due to gravity in m s^{-2};
- h is the depth at which the pressure is being calculated in m.

After reading this report, a student investigates the relationship between pressure due to a liquid and the depth of the liquid. The results obtained are presented in the following graph.



Use the gradient of the graph and the relationship given above to calculate the density of the liquid which the student used.

11.2 Course style questions

Course style questions

Q6:

1. State the difference between a scalar quantity and a vector quantity.

(1)

2. A dog-team and sledge travels 10 km on a bearing of 090 in a time of 20 minutes. The dog-team and sledge then turns on to a bearing of 120 and travels a further 14 km in a time of 35 minutes.

   a) By scale drawing or otherwise, find the displacement of the dog-team and sledge during this journey.

(3)

   b) Calculate the average velocity, in m s-1, of the dog-team and sledge during this journey.

(3)

3. At one point of the journey, where the ground is horizontal, the dog-team applies a 95 N force to the sledge of mass 120 kg. The frictional forces acting on the sledge are 25 N.

   a) Show that the acceleration of the sledge will be 0.58 m s^{-2}.

(2)

   b) This acceleration is maintained for a time of 20 s and the final velocity of the sledge is 14 m s^{-1}.

   Calculate the velocity of the sledge at the start of the acceleration.

(3)

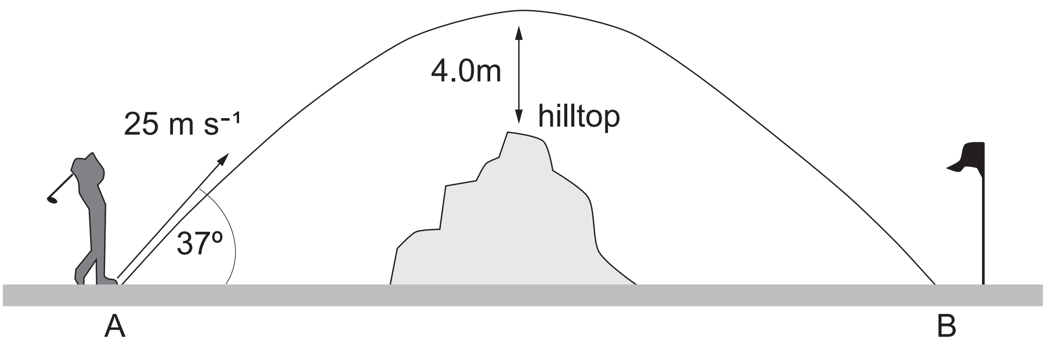
Marks (12)

. . . .

Q7: A golfer hits a golf ball from point A. The ball leaves the club with an initial velocity of 25 m s^{-1} at an angle of 37° to the horizontal.

The ball travels through the air and lands at point B.

At the ball's highest point it clears a hilltop by a distance of 4.0 m.



1. a) Calculate the horizontal component of the initial velocity of the ball.

(1)

b) Calculate the vertical component of the initial velocity of the ball.

(1)

c) Calculate the height of the highest point of the hill.

(4)

2. a) Sketch a graph that shows how the horizontal component of the velocity varies during the first 2 seconds. Numerical values are required on both axes.

(2)

b) Sketch a graph that shows how the vertical component of the velocity varies during the first 2 seconds. Numerical values are required on both axes.

(2)

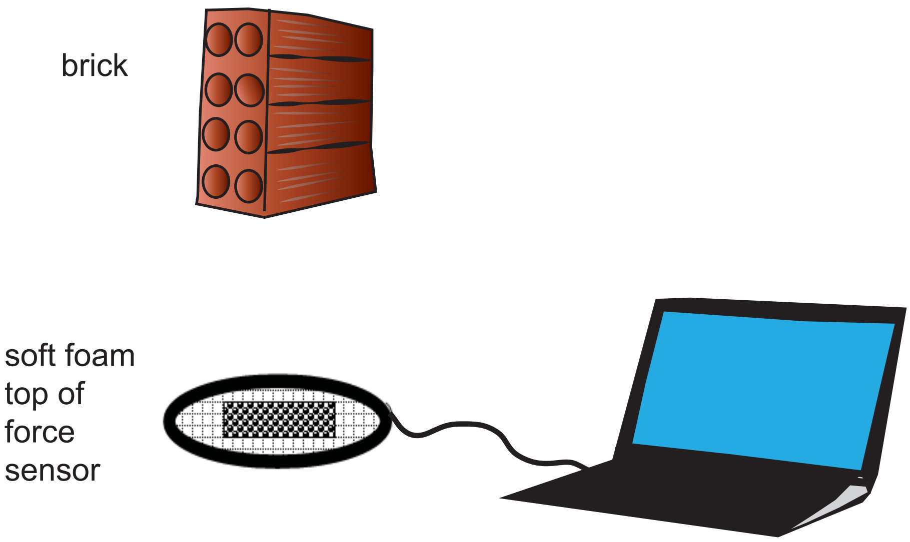
3. When the effects of air resistance are not ignored, the golf ball follows a different path.
Suggest a possible value for the height that the ball clears the hilltop.
You must justify your answer.

(2)

Marks (12)

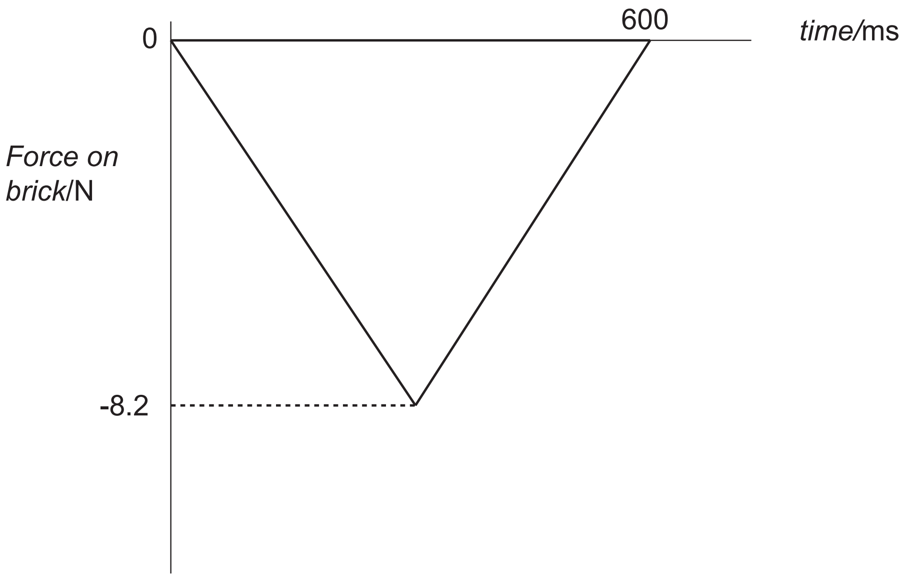
. . . .

Q8: A piece of soft foam is placed on top of a force sensor. The force sensor is connected to a computer.



A brick is dropped from a height onto the piece of soft foam. The brick stops without rebounding from the foam.

The computer displays the following force-time graph for the collision between the falling brick and the foam.



a) Calculate the impulse on the brick during the collision.

(3)

b) State the direction of the impulse on the brick.

(1)

c) The velocity of the brick as it hit the foam was 7.6 m s^{-1}. Calculate the mass of the brick.

(3)

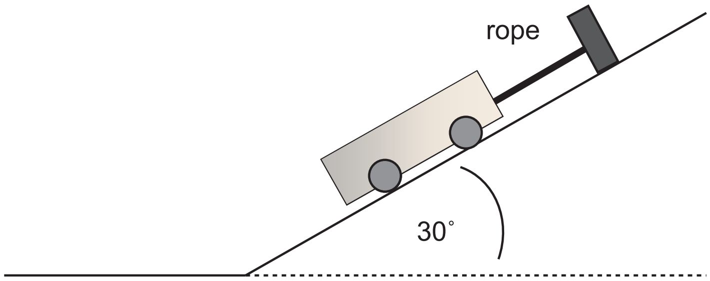
d) The soft foam is replaced by a piece of harder foam. As before the brick is dropped and the velocity of the brick as it hits the harder foam is again 7.6 m s^{-1}. Again the brick stops without rebounding.
Explain how the stopping time of the brick will change when the harder foam is used.

(2)

Marks (9)

. . . .

Q9: A trolley of mass 2.0 kg is held at rest on a slope using a rope. The slope is at an angle of 30° to the horizontal.



a) Draw a sketch showing all the forces acting parallel to the slope on the trolley.

(1)

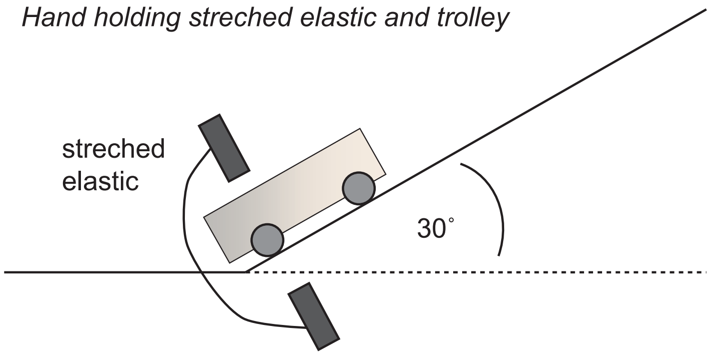
b) Calculate the component of the weight of the trolley acting parallel to the slope.

(3)

c) The rope is cut and the trolley accelerates down the slope at 1.5 m s^{-2}. Calculate the frictional force acting on the trolley.

(4)

d) The same slope and trolley are used along with a stretched elastic to cause the trolley to move up the slope.



The elastic and trolley are released and the trolley is allowed to move freely up the slope. The table shows the acceleration of the trolley as it moves down and then up the slope.

Direction of movement of trolley	*Acceleration down slope* / m s^{-2}
down slope	1.5
up slope	8.3

Explain, in terms of the forces acting on the trolley, why the acceleration of the trolley down the slope is greater when it is moving up the slope.

(2)

Marks (10)

. . . .

Q10: Alpha Centauri, a nearby star in our galaxy, is a distance of 4.3 light-years from Earth.

A rocket leaves Earth for Alpha Centauri at a speed of 0.95c relative to an observer on Earth. Assume that the Earth and Alpha Centauri are stationary with respect to one another.

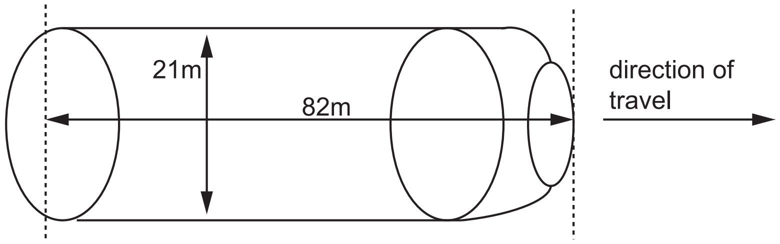
1. a) The astronauts measure the time, in years, for this journey. Calculate the value that they measure.

   (3)

   b) Show by calculation the distance the astronauts would measure for this journey.

   (3)

   c) One of the astronauts, using a meter stick, measures the length and diameter of the cylindrical spacecraft to be 82 and 21 m, as shown.

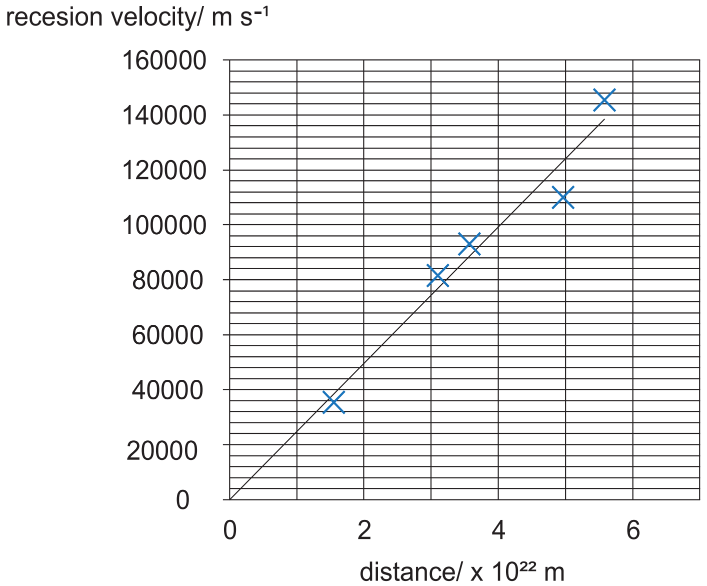
   Assume that the spacecraft is oriented with its long cylindrical axis in the direction of motion.
   Explain which measurement, the length or diameter, is in agreement with the measurement taken by an observer on Earth.

   (1)

2. a) State what evidence led Hubble to propose that galaxies are receding from each other.

   (2)

   b) The graph shows recession velocity of galaxy against distance to galaxy.

   Based on the graph, calculate the best estimate of Hubble's constant.

   (3)

Marks (12)

11.3 End of unit assessment

End of unit 1 test Go online

The following data should be used when required:

acceleration due to gravity g	*9.8 m* s^{-2}
speed of sound	*340 m* s^{-1}
universal constant of gravitation G	*6.67 x* 10^{-11} m^3 kg^{-1} s^{-2}
mass of the Earth	*5.97 x* 10^{24} *kg*
mass of the Moon	*7.35 x* 10^{22} *kg*
radius of the Earth	*6.38 x* 10^{6} *m*
radius of the Moon	*1.74 x* 10^{6} *m*

A reminder of this useful data values can be found in the information sheet (opened by clicking within a test).

The end of unit test is available online. If however you do not have access to the web you may try the questions which follow.

Q11: In the equation s = ut + $^1/_2$ at ^{2}, the term *ut* represents the

a) initial acceleration
b) displacement when the acceleration is zero
c) acceleration after t seconds
d) initial velocity
e) velocity after t seconds

. . . .

Q12: A ball is thrown horizontally from the window of a building with velocity 14 m s^{-1}. The window is 24 m above the ground.

Calculate the horizontal distance (in m) between the building and the point where the ball hits the ground.

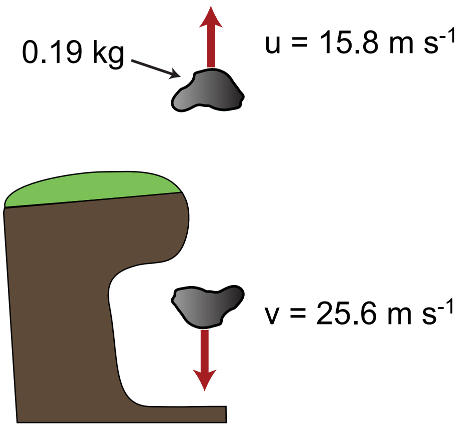
. . . .

Q13: A stone dropped from the roof of a building hits the ground travelling at a speed of 20.5 ms $^{-1}$.

How tall is the building, in m?

. . . .

Q14: A stone of mass 0.19 kg is thrown vertically upwards with a speed of 15.8 m s^{-1} from the edge of a cliff as shown in the diagram.

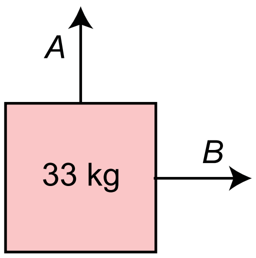


The stone lands with a speed of 25.6 m s^{-1} at the bottom of the cliff. The effects of air resistance on the stone throughout its motion are negligible.

1. Calculate the time (in seconds) that the stone takes to reach its highest point.
2. What is the total time (in seconds) that the stone is in the air?
3. Calculate, in joules, the change in the kinetic energy of the stone during its motion.
4. Use the previous answer and calculate in metres the height, h , of the thrower's hand above the bottom of the cliff.

. . . .

Q15: The diagram shows a top view of two perpendicular forces A and B which are being applied to a crate of mass 33 kg, in order to slide the crate along a horizontal surface.

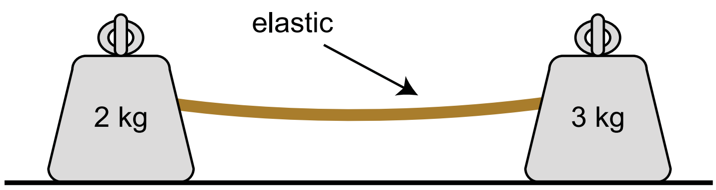


The magnitude of A is 5.0 N and the magnitude of B is 12 N.

1. Calculate the size of the resultant force due to A and B , in N.
2. As the crate slides in the direction of the resultant of A and B , a constant frictional force of 8.7 N opposes the motion. Calculate the magnitude of the acceleration of the crate, in m s^{-2}. (State this answer to 3 significant figures as you will be using it in the final part of the question.)
3. Calculate the distance moved (in m) by the crate in the first 5.0 s of its motion.

. . . .

Q16: Two masses of 2 kg and 3 kg are joined by a long piece of elastic and are then moved apart on a smooth horizontal bench until the elastic is stretched considerably.



The masses are released simultaneously.

When the 3 kg mass has moved a distance d, the 2 kg mass has moved a distance

a) 0.67 d
b) 0.60 d
c) 0.55 d
d) 0.40 d
e) 1.5 d

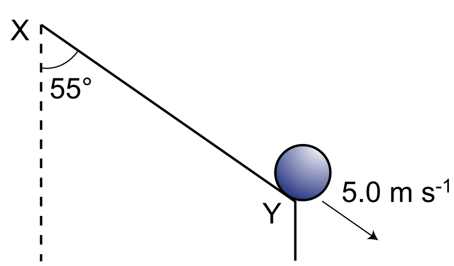
. . . .

Q17: A cart of mass 50 kg travelling at 3.6 m s $^{-1}$ rolls head-on into a stationary cart of mass 70 kg. After the collision, the carts couple together. Assume no external forces are acting on the carts before or during the collision.

1. Calculate the velocity in m s $^{-1}$ of the two carts immediately after the collision.
2. Calculate the amount of kinetic energy lost in the collision, in J.
3. After the collision a constant force of 12 N opposes the motion of the carts. Calculate the total distance the carts travel after the collision before they come to rest, giving your answer in m.

. . . .

Q18: A ball rolls down a ramp XY, which is inclined at 55 ° to the vertical, and leaves the ramp at a velocity of 5.0 m s $^{-1}$.

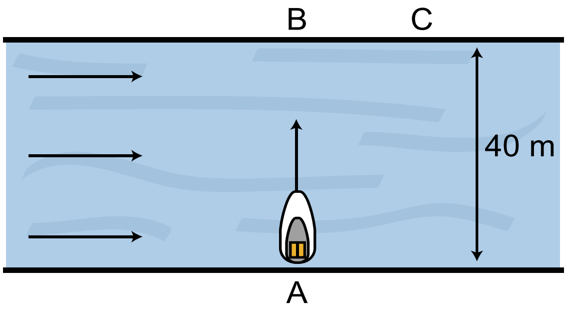


1.5 s after the ball leaves Y the vertical component of velocity of the ball, in m s $^{-1}$, is given by

a) (5.0 sin 55° + 14.7)
b) (5.0 + 9.8)
c) (5.0 cos 55° + 9.8)
d) (5.0 sin 55° + 9.8)
e) (5.0 cos 55° + 14.7)

. . . .

Q19: A speed boat is crossing a river. The boat starts from point A, and the driver points the boat at point B, directly across the river from point A. The river is 40 m wide.

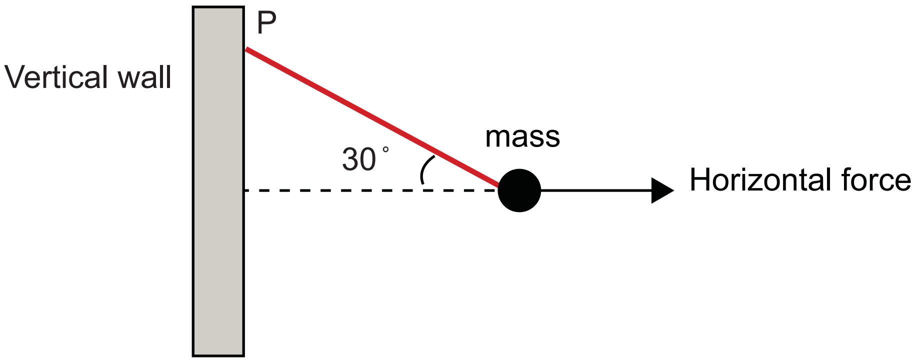


The boat has velocity 7.6 m s $^{-1}$ in the direction AB. The river is flowing at velocity 3.2 m s $^{-1}$, so that the boat actually arrives at point C, downstream of point B.

1. Calculate the magnitude of the velocity in m s $^{-1}$ of the boat in the direction AC.
2. Calculate the distance BC, in m.

. . . .

Q20: A string is attached to a vertical wall at position P. A 1.9 kg mass is attached to the other end of the string.



A horizontal force is applied to the mass so that the string is held stationary in the position shown.

Calculate the tension in the string.

. . . .

Q21: On a certain planet, the gravitational force acting on a mass of 2.0 kg is 24 N. The gravitational field strength on the planet is

a) 24 N kg $^{-1}$
b) 10 N kg $^{-1}$
c) 12 N kg $^{-1}$
d) 6.0 N kg $^{-1}$
e) 22 N kg $^{-1}$

. . . .

Q22: A satellite of mass 1200 kg is orbiting the earth at a height of 5.45×10^5 m above the earth's surface.

Calculate the gravitational force (in N) that the earth exerts on the satellite.

....

Q23: A distant planet has mass 5.45×0^{25} kg. A moon, mass 4.04×10^{22} kg orbits this planet with an orbit radius of 7.36×10^8 m.

Calculate the size of the gravitational force that exists between the moon and the planet, in N.

....

Q24: Two identical solid spheres each have mass 0.803 kg and diameter 0.225 m.

Find the gravitational force, in N, between them when they are touching.

....

Q25: Given that the mass of the planet Neptune is 1.03×10^{26} kg and its radius is 2.48×10^7 m, calculate the weight, in N, of a 5.64 kg mass on the surface of Neptune.

....

Q26: On the surface of the Earth, a particular object has a weight of 23 N.

Calculate its weight on the surface of the Moon, in N.

....

Q27: State the expression for the apparent frequency f_o detected when a source of sound waves of frequency f_s moves towards a stationary observer at a speed v_s .

....

Q28: A male bat flies towards his mate at a speed of 20.0 m s^{-1} while emitting ultrasound of frequency 28.5 kHz.

Calculate the apparent frequency of the ultrasound detected by the stationary female, giving your answer in kHz.

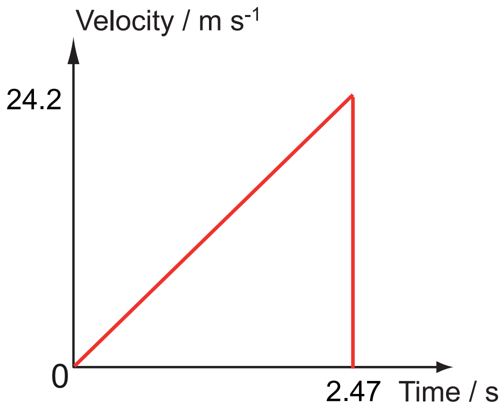
....

Q29: A cyclist, travelling at a steady speed of 8 m s^{-1}, approaches and passes a brass band. A trumpeter in the band is playing series of notes of frequency 512 Hz.

Calculate the difference between the frequencies of the notes played by the trumpeter as observed by the cyclist when approaching, and having passed the band.

....

Q30: An object is dropped from a tall building. The following velocity-time graph is obtained:



1. Calculate the acceleration of the object in m s $^{-2}$
2. Calculate the height of the building in metres.

. . . .

Q31: A car has a mass of 1000 kg. This car accelerates from rest to 20 m s $^{-1}$ in 40 seconds.

1. Calculate the kinetic energy of the car, in J, when it reaches its final velocity.
2. Calculate the average power, in W, of the car's engine.
3. The car is now brought to a halt by the brakes of the car in a distance of 25 m. Calculate the average force, in N, of the car's brakes.

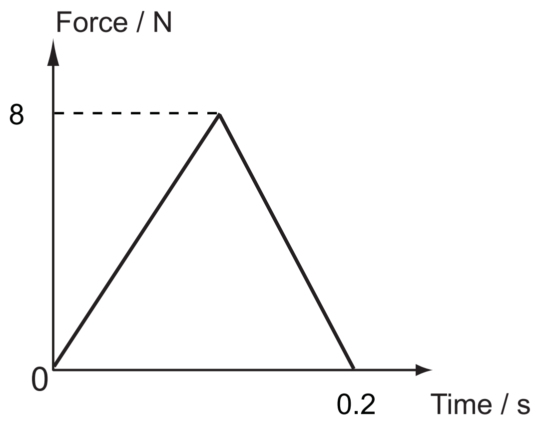
. . . .

Q32: A child of mass 20 kg is wearing roller skates is stationary on a smooth surface. The child throws a ball of mass 0.5 kg horizontally away from themself at a speed of 4.0 m s $^{-1}$.

1. Calculate the momentum of the ball in kg m s $^{-1}$
2. Calculate the resultant momentum of the child in kg m s $^{-1}$.
3. Calculate the resultant velocity of the child in m s $^{-1}$.

. . . .

Q33: When a putter strikes a stationary golf ball the following force time graph is obtained.



1. Calculate the impulse on the ball in Ns.
2. What is the change in momentum of the ball in kg m s $^{-1}$?
3. If the ball has a mass of 0.02 kg what is its velocity in m s $^{-1}$ after being struck?

. . . .

Q34: A spacecraft is travelling at a constant speed of 50 % of the speed of light. Calculate the length contraction in metres for a stationary observer if an observer on the spacecraft measures the length of the spacecraft to be 20 metres.

Appendix A

Units, prefixes and scientific notation

Contents

A.1 Symbols and units used in Unit 1: Our dynamic universe 216
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A.1 Symbols and units used in Unit 1: Our dynamic universe

Physics Quantity	Symbol	Unit	Unit Abbreviation
distance	d	metre	m
displacement	s	metre	m
speed, velocity	v	metre per second	m s^{-1}
average velocity	$\bar{v}$	metre per second	m s^{-1}
change in velocity	Δv	metre per second	m s^{-1}
initial velocity	u	metre per second	m s^{-1}
final velocity	v	metre per second	m s^{-1}
acceleration	a	metre per second square	m s^{-2}
time	t	second	s
mass	m	kilogram	kg
weight	W	newton	N
force	F	newton	N
energy	E	joule	J
work done	E_w	joule	J
potential energy	E_p	joule	J
acceleration due to gravity	g	metre per second square	m s^{-2}

Physics Quantity	Symbol	Unit	Unit Abbreviation
acceleration due to gravity	g	metre per second square	m s^{-2}
gravitational field strength	g	newton per kilogram	N kg^{-1}
height	h	metre	m
kinetic energy	E_k	joule	J
power	P	watt	W
momentum	p	kilogram metre per second	kg m s^{-1}
impulse	—	newton second	N s
universal constant of gravitation	G	metre cube per (kilogram second square)	m^3 kg^{-1} s^{-2}
distance between two point masses	r	metre	m
speed of light in a vacuum	c	metre per second	m s^{-1}
observed time (stationary observer)	t'	second	s
observed time (moving observer)	t	second	s
observed length (stationary observer)	l'	metre	m
observed length (moving observer)	l	metre	m
observed frequency (stationary observer)	f_o	hertz	Hz
source frequency (moving source)	f_s	hertz	Hz
velocity of moving source	v_s	metre per second	m s^{-1}
wavelength measured (source stationary)	λ_{rest}	metre	m

Physics Quantity	**Symbol**	**Unit**	**Unit Abbreviation**
wavelength measured (source moving)	$\lambda_{observed}$	metre	m
red shift	z	- - -	- - -
Hubble constant	H_O	per second	s^{-1}

A.2 Significant figures

It is important when calculating numerical values that the final answer is quoted to an appropriate number of significant figures.
As a general rule, the final numerical answer that you quote should be to the same number of significant figures as the data given in the question.

The above rule is the key point but you might like to note the following points:

1. The answer to a calculation cannot increase the number of significant figures that you can quote.
2. If the data is not all given to the same number of significant figures, identify the least number of significant figures quoted in the data. This least number is the number of significant figures that your answer should be quoted to.
3. When carrying out sequential calculations carry many significant figures as you work through the calculations. At the end of the calculation, round the answer to an appropriate number of significant figures.
4. In the Higher Physics course quoting an answer to three significant figures will usually be acceptable.

Examples

1. The current in a circuit is 6.7 A and the voltage across the circuit is 21 V. Calculate the resistance of the circuit.

Note: Both of these pieces of data are given to two sig. figs. so your answer must also be given to two sig figs.

I = 6.7 A
V = 21 V
R = ?

$$V = I\,R$$
$$21 = 6.7 \times R$$
$$R = 3.1343$$
$$R = 3.1\ \Omega$$

round to 2 sig figs

. . . .

2. A 5.7 kg mass accelerates at 4.36 m s^{-2}.
Calculate the unbalanced force acting on the mass.

Note: The mass is quoted to two sig. figs and the acceleration is quoted to three sig. figs. so the answer should be quoted to two sig figs.

m = 5.7kg
a = 4.36 m s^{-2}
F = ?

$F = m\ a$

$F = 5.7 \times 4.36$

$F = 24.852$

$F = 25\ N$

round to 2 sig figs

....

3. A car accelerates from 0.5037 m s^{-1} to 1.274 m s^{-1} in a time of 4.25 s.
The mass of the car is 0.2607 kg.
Calculate the unbalanced force acting on the car.

Note: The time has the least number of sig figs, three, so the answer should be quoted to three sig figs.

u = 0.5037 m s^{-1}
v = 1.274 m s^{-1}
t = 4.25 s
m = 0.2607 kg

Step 1: calculate a

$$a = \frac{v - u}{t}$$

$$a = \frac{1.274 - 0.5037}{4.25}$$

$$a = 0.181247\ m\ s^{-2}$$

Step 2: calculate F

$F = m\ a$

$F = 0.2607 \times 0.18147$

$F = 0.0472511$

$F = 0.0473\ N$

round to 3 sig figs

Quiz questions Go online

Q1: A car travels a distance of 12 m in a time of 9.0 s.
The average speed of the car is:

a) 1.3333
b) 1.33
c) 1.3
d) 1.4
e) 1

. . . .

Q2: A mass of 2.26 kg is lifted a height of 1.75 m. The acceleration due to gravity is 9.8 m s^{-2}.
The potential energy gained by the mass is:

a) 38.759 J
b) 38.76 J
c) 38.8 J
d) 39 J
e) 40 J

. . . .

Q3: A trolley of 5.034 kg is moving at a velocity of 4.03 m s^{-1}.
The kinetic energy of the trolley is:

a) 40.878 J
b) 40.88 J
c) 40.9 J
d) 41 J
e) 40 J

A.3 Scientific notation

When carrying out calculations, you should be able to use scientific notation. This type of notation has been used throughout the topics where necessary, so you will already be familiar with it

Remember scientific notation is used when writing very large or very small numbers. When a number is written in scientific notation there is usually one, nonzero number, before the decimal point.

Examples

1. The speed of light is often written as 3 x 10^{8} m s^{-1}.

This can be converted into a number in ordinary form by moving the decimal point 8 places to the right, giving 300 000 000 m s^{-1}.

....

2. The capacitance of a capacitor may be 0.000 022 F.

This very small number would often be written as 2.2 x 10^{-5} F. The x 10^{-5} means move the decimal point 5 places to the left.

Make sure you know how to enter numbers written in scientific notation into your calculator.

A.3.1 Prefixes

There are some prefixes that you must know. These are listed in the following table:

Prefix	Symbol	Symbol
pico	p	$\times 10^{-12}$
nano	n	$\times 10^{-9}$
micro	μ	$\times 10^{-6}$
milli	m	$\times 10^{-3}$
kilo	k	$\times 10^{3}$
mega	M	$\times 10^{6}$
giga	G	$\times 10^{9}$

In Higher Physics you are expected to know and remember the meaning of all of these prefixes.

Glossary

Blueshift

Doppler-shifting of a light wave towards the blue end of the spectrum (observed frequency higher than emitted frequency) owing to relative motion of the source towards the observer

Components of a vector

two vectors which act at right angles, the vector sum of which is the original vector

Conservation of energy

energy cannot be created or destroyed, only converted from one form to another

Conservation of momentum

when two or more objects interact, the total momentum is conserved, in the absence of external forces

Dark energy

a theoretical form of energy postulated to act in opposition to gravity and to occupy the entire universe, accounting for most of the energy in it and causing its expansion to accelerate

Dark matter

thought to be a type of matter which does not interact with electromagnetic radiation so is invisible to astronomers detecting light or any other type of electromagnetic radiation eg radio to gamma

Displacement

a specified distance from a fixed point, in a specified direction. Displacement is a vector quantity.

Doppler effect

the observed change in frequency of a wave caused by relative motion between the source and observer

Elastic collision

a collision in which both momentum and kinetic energy are conserved

Gravitational field

the region of space around an object in which any other object with a mass will have a gravitational force exerted on it by the first object

Gravitational field strength

the gravitational field strength at a point in a gravitational field is equal to the force acting per unit mass placed at that point in the field

Hubble's law

Hubble's law states that a galaxy's velocity is proportional to the distance from the observer. Mathematically $v = h_o d$ where h_o is the Hubble constant.

Impulse

the change of momentum of an object, equal to the product of the force acting on the object and the time over which the force acts

Inelastic collision

a collision in which momentum is conserved but kinetic energy is not

Kinetic energy

the energy of an object due to its motion

Momentum

the product of the mass of an object and its velocity. Momentum is a vector quantity, measured in kg m s^{-1}.

Newton

one newton is the force that, when applied to an object of mass 1 kg, will cause the object to accelerate at a rate of 1 m s^{-2} in the direction of the applied force

Potential energy

the energy stored in an object due to its position, its shape or its state

Projectile

a projectile is an object that is flying through the air under the influence of gravity. This means that the object is moving in two dimensions.

Redshift

Doppler-shifting of a light wave towards the red end of the spectrum (observed frequency lower than emitted frequency) owing to relative motion of the source away from the observer

Scalar

a physical quantity which has magnitude but no direction

Universal Law of Gravitation

also known as Newton's law of gravitation, this law states that there is a force of attraction between any two massive objects in the universe. For two point objects with masses m_1 and m_2, placed a distance r apart, the size of the force F is given by the equation $F = \frac{Gm_1m_2}{r^2}$

Vector

a physical quantity which has direction as well as magnitude

Velocity

the rate of change of displacement. Velocity is a vector quantity.

Weight

the gravitational force acting on a mass

Hints for activities

Topic 1: Motion: equations and graphs

Quiz: Components of a vector

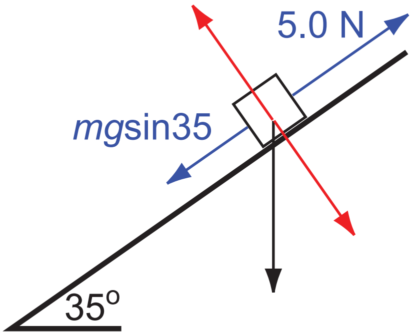
Hint 1: The angle θ given is between the vector and the vertical - refer to the section titled Components of a vector - the vertical component of each force = F cos θ. The angles and the forces are equal - so the total vertical force is double the vertical component of one force.

Topic 2: Forces, energy and power

Mass on a slope

Hint 1:

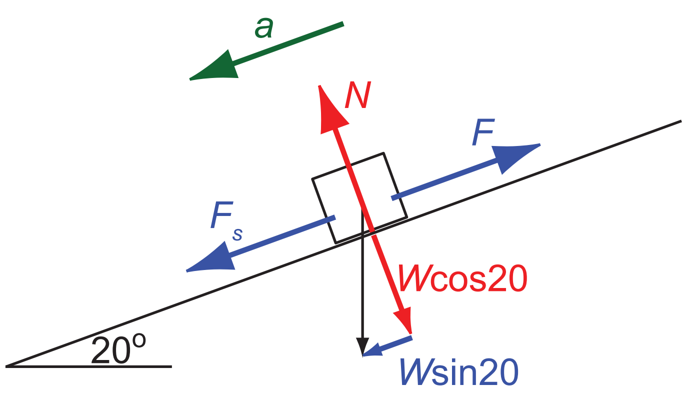
The free body diagram is as follows, with the forces acting parallel to the slope in blue, and those acting perpendicular to the slope in red:



Quiz: Free body diagrams

Hint 1:

The velocity of the crate is constant - what does this tell you about the forces acting on the crate? If you are not sure how to calculate the component of the weight of the crate acting down the slope, see the following image or refer to the section titled Objects undergoing acceleration



Free body diagram of the television sliding down the slope

Hint 2:

First use $F = ma$ to find the unbalanced vertical force acting on the block - then consider all the vertical forces to find the tension in the rope

Quiz: Energy and power

Hint 1:

Use the equation $E_k = \frac{1}{2}mv^2$ or refer to the section titled Energy.

Hint 2:

The potential energy of the tile is converted to kinetic energy as it falls - see the following equations or refer to the section titled Energy.

$$E_k = \frac{1}{2}mv^2$$

$$E_p = mgh$$

Hint 3:

Use the equation $P = F \times v$ or refer to the section titled Power

Hint 4:

Long method: find the maximum height - then find the potential energy at that height - subtract this from the initial kinetic energy.

Shorter method: find the horizontal component of the initial velocity of the projectile. At the maximum height the velocity of the projectile is horizontal so you can use this value in the kinetic energy equation to find the kinetic energy at the maximum height.

Topic 3: Collisions, explosions and impulse

Quiz: Momentum

Hint 1:

All of ball *M*'s momentum is transferred to ball *N*.

Hint 2:

The carriages have the same velocity after the collision.

Hint 3:

See the section titled Inelastic collisions, for the definition of an inelastic collision

Hint 4:

Refer to these equations $E_k = \frac{1}{2}mv^2$ and $p = mv$ or look at the sections titled Energy and Momentum.

Hint 5:

Let the velocities of spheres after the collision be v_1 and v_2.

Use conservation of momentum to find a linear equation in v_1 and v_2 .

Use conservation of kinetic energy to find a quadratic equation in v_1 and v_2.

Use simultaneous equations to find the values of v_1 and v_2.

You will get two possible answers - one of these is not physically possible as it requires the 1.0 kg sphere to pass through the 4.0 kg sphere.

Quiz: Impulse

Hint 1:

See section titled Impulse for a definition of impulse

Hint 2:

See section titled Impulse

Hint 3:

The words '*at rest*' mean the initial momentum is 0 kg m s^{-1}.

Hint 4:

Quantities that have the same dimensions have units which are equivalent.

Hint 5:

The impulse *Ft* is equal to the change in momentum of the car.

Topic 7: The expanding universe

Quiz: Doppler effect

Hint 1: See the section titled The Doppler effect with a moving source.

Hint 2: See the section titled The Doppler effect with a moving source for the correct relationship.

Hint 3: See the section titled The Doppler effect with a moving observer.

Hint 4: The observed frequency is higher if the source and observer are getting closer together.

Hint 5: See the section titled The Doppler effect with a moving source for the correct relationship.

Appendix A: Units, prefixes and scientific notation

Quiz questions

Hint 1: Data is quoted to 2 sig figs so answer must be quoted to 2 sig figs.

Hint 2: The acceleration due to gravity is quoted to only 2 sig figs so the answer must be given to 2 sig figs.

Hint 3: The mass of the trolley is given to 4 sig figs and the velocity is given to 3 sig figs.

Answers to questions and activities

Topic 1: Motion: equations and graphs

Distance and displacement (page 5)

Q1: D

Q2: B

Q3: E

Q4: E

Adding collinear vectors (page 7)

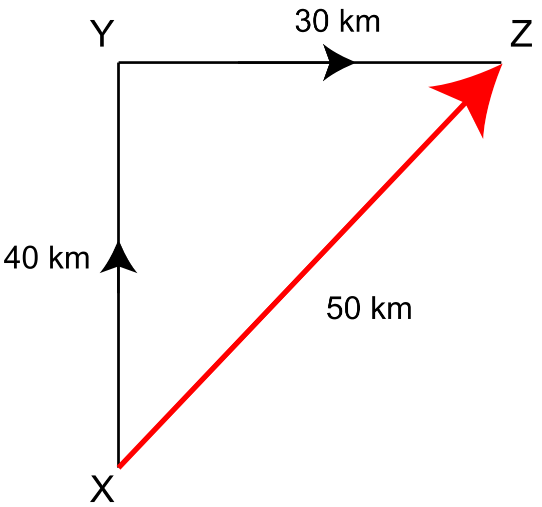
Q5: +40

Q6: -40

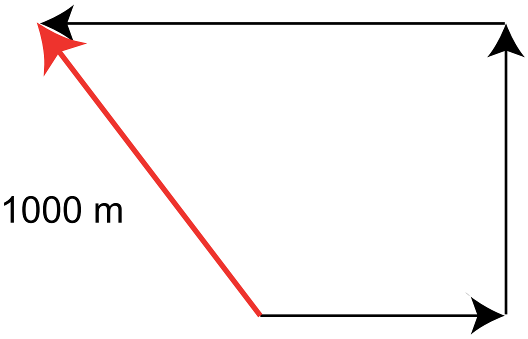
Q7: -40

Crossing the river (page 9)

Q8:

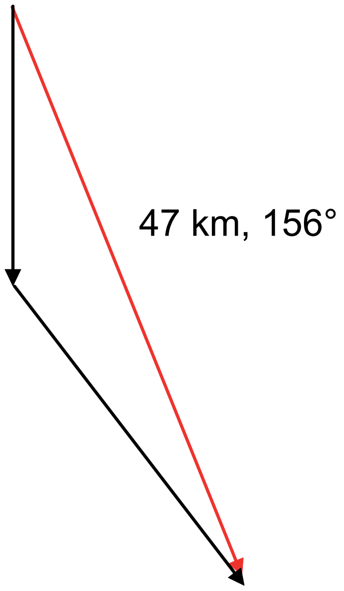


Q9:

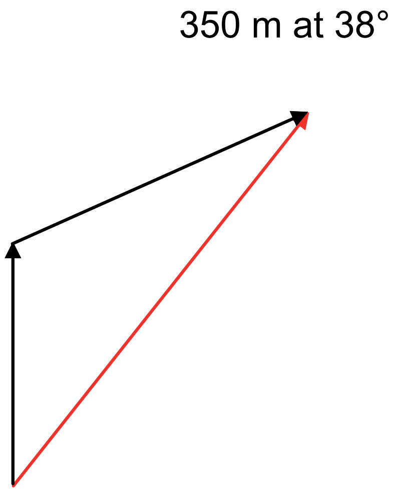


Addition of vectors (page 10)

Q10:



Q11:



Quiz: Adding vectors (page 11)

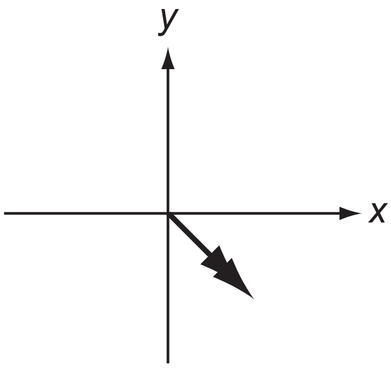
Q12: e) 115 N

Q13: b) 30 N acting southwards

Q14: d) 139 N

Q15: b) 30°

Q16: d)



Quiz: Components of a vector (page 16)

Q17: c) 245 m s^{-1}

Q18: b) -3.06

Q19: d) +10.2

Q20: d) 83 N

Q21: e) 640 N

Horizontal Motion (page 28)

Expected answer

List the data you are given in the question:

u = 12.0 m s^{-1}, v = 0 m s^{-1}, s = 30.0 m, a = ?

The appropriate kinematic relationship is

$$v^2 = u^2 + 2as$$

Putting the values into this equation,

$$\begin{aligned} v^2 &= u^2 + 2as \\ \therefore 0^2 &= 12.0^2 + (2 \times a \times 30.0) \\ \therefore 0 &= 144 + 60a \\ \therefore -60a &= 144 \\ \therefore a &= -\frac{144}{60} \\ \therefore a &= -2.40 \text{ m s}^{-1} \end{aligned}$$

So to stop the car in exactly 10.0 m, the car must have an acceleration of -2.40 m s^{-2}, equivalent to a deceleration of 2.40 m s^{-2}.

Quiz: Kinematic relationships (page 28)

Q22: b) 28 m

Q23: e) 9.8 m s^{-2} downwards

Q24: c) 33 m s^{-1}

Q25: c) -4.2 m s^{-2}

Q26: d) 27 m

Extra Help: Interpretation of graphs (page 33)

Q27: 10

Q28: 10

Q29: constant

Q30: 5

Q31: 0

Q32: 0

Q33: constant

Q34: 0

Q35: -20

Q36: -20

Q37: constant

Q38: -10

Q39: 0

Q40: 0

Q41: constant

Q42: 0

Q43: 15

Q44: 5

Q45: acceleration

Q46: -10

Q47: -5

Q48: -15

Q49: acceleration

Q50: -10

Q51: -15

Q52: -25

Q53: acceleration

Q54: -10

Motion of a bouncing ball (page 42)

Expected answer

1. Starting from a height of 10 m, the data is *u* = 0 m s^{-1}, *a* = -9.8 m s^{-2}, *s* = -10 m and *v* is unknown. The kinematic relationship that should be used is $v^2 = u^2 + 2as$

$$\begin{aligned} v^2 &= u^2 + 2as \\ \therefore v^2 &= 0 + (2 \times -9.8 \times -10) \\ \therefore v^2 &= 196 \\ \therefore v &= -14 \text{ m s}^{-1} \end{aligned}$$

   Note that we have taken the *negative* square root - if the ball is moving downwards, its velocity is negative in the sign convention we are using.

2. Now the ball is projected upwards with *u* = 12 m s^{-1}, *v* = 0 m s^{-1}, *a* = -9.8 m s^{-2} and *s* is unknown.

$$\begin{aligned} v^2 &= u^2 + 2as \\ \therefore 0 &= 12^2 + (2 \times -9.8 \times s) \\ \therefore 19.6s &= 144 \\ \therefore s &= 7.3 \text{ m} \end{aligned}$$

3. Velocity is negative so ball is moving downwards.
4. Velocity is positive so ball is moving upwards.
5. Whenever the ball's velocity changes sign it changes direction of motion. This is because velocity is a vector quantity and if there are velocities in opposite directions then one must be positive and one must be negative.

Please note: when the ball is in contact with the ground, *s* = 0 m, the ball experiences a large upward (so positive) acceleration which lasts for a very short time.

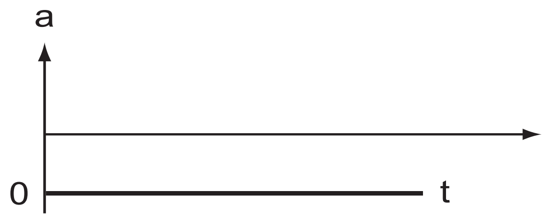
Quiz: Acceleration (page 43)

Q55: d) 3.6 m s^{-2}

Q56: b) The velocity of the motorcycle is constant, so its acceleration is zero.

Q57: a) 0.4 m s^{-2}

Q58: e)



Q59: b) A and E

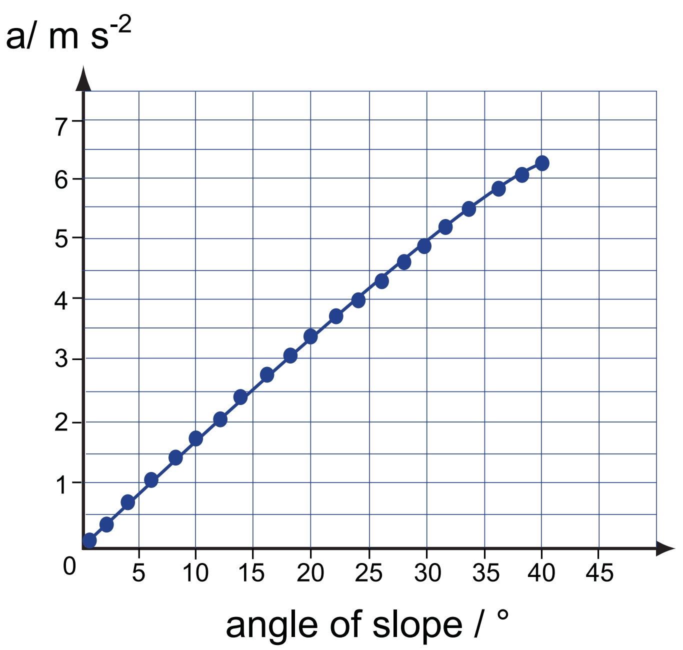
Freefall (page 47)

Q60: g = -9.8 m s^{-2}

Acceleration on a slope (page 50)

Q61: The results data is given in the following table:

angle°	acceleration/m s^{-2}
0	0
2	0.342015068
4	0.683613443
6	1.02437894
8	1.363896389
10	1.701752141
12	2.03753457
14	2.370834577
16	2.701246087
18	3.028366545
20	3.351797405
22	3.671144615
24	3.986019102
26	4.296037239
28	4.600821315
30	4.9
32	5.193208789
34	5.480090454
36	5.760295472
38	6.033482458
40	6.299318575



1. As the angle increases so does the acceleration.
2. No. The graph is not a straight line through the origin. For small angles it appears to be but as the angles become larger the graph starts to flatten out so the two quantities are not directly proportional.

End of topic 1 test (page 52)

Q62: -0.5 m s $^{-2}$

Q63: 13.1 m

Q64: 29.7 m

Q65: 17.4 m s $^{-1}$

Q66:

1. 2.75 s
2. 20.8 m s $^{-1}$

Q67: 54 N

Q68: 42 N

Q69:

1. 50 N
2. -25 N

Q70: 601 N

Q71: c) Displacement and force

Q72: 64 N

Q73:

1. 8.5 m s $^{-1}$
2. 17 m

Q74:

1. -4.9 N
2. 12.6 N

Q75:

1. 0.25 m s $^{-2}$
2. 50 m
3. 0.45 m s $^{-2}$
4. 0 m s $^{-2}$

Q76:

1. 8 m s^{-1}
2. 10 m s^{-1}

Q77:

1. 3.2 m s^{-2}
2. 2.05 m s^{-2}

Q78: 6 m s^{-1}

Q79: -9.8 m s^{-2}

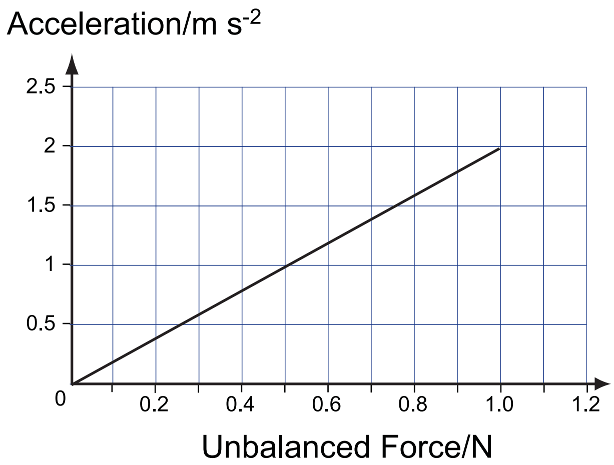
Q80:

1. It increases
2. No

Topic 2: Forces, energy and power

Newton's second law (page 62)

Q1:



Q2: As the unbalanced force is increased the acceleration increases.

Since the graph is a straight line through the origin, the acceleration of an object is directly proportional to the unbalanced force acting on it

Quiz: Newton's second law (page 64)

Q3: c) 1 kg m s^{-2}

Q4: b) 15 m s^{-2}

Q5: b) 0.18 m s^{-2}

Q6: e) 4*a*

Q7: d) 25 m s^{-1}

Mass on a slope (page 80)

Expected answer

Using the free body diagram, the unbalanced force acting down the slope is $mg\sin\theta$ - 5.0. We can use this in Newton's second law:

$$\begin{aligned} mg\sin\theta - 5.0 &= ma \\ \therefore a &= \frac{mg\sin\theta - 5.0}{m} \\ \therefore a &= \frac{(4.0 \times 9.8 \times 0.574) - 5.0}{4.0} \\ \therefore a &= \frac{22.5 - 5.0}{4.0} \\ \therefore a &= 4.4 \text{ m s}^{-2} \end{aligned}$$

Quiz: Free body diagrams (page 82)

Q8: d) In the direction of *D*

Q9: a) 62 N

Q10: c) 41 N

Q11: d) 4.4 N $\Leftarrow$

Q12: e) 34 N

Quiz: Energy and power (page 89)

Q13: d) 360 J

Q14: b) 14 m s^{-1}

Q15: d) 3000 W

Q16: c) 970 J

End of topic 2 test (page 91)

Q17: 4.94 N

Q18: 216 N

Q19: 31.25 m

Q20:

1. 1.52 m s^{-2}
2. 3.5 N

Q21:

1. 7840 N
2. 8008 N

Q22: 4.03 m s^{-2}

Q23:

1. 93.3 N
2. 76.2 N

Q24: 5.47 m s^{-1}

Q25:

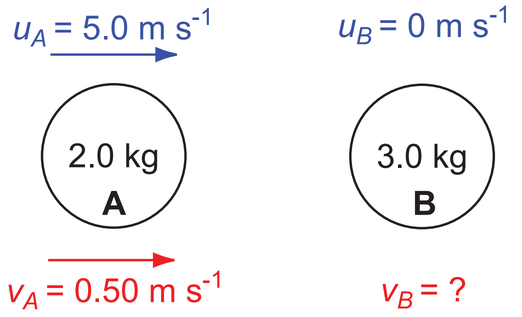
1. 1.71 m s^{-1}
2. 18.7 J

Q26: 811 J

Topic 3: Collisions, explosions and impulse

Collision between two spheres (page 99)

Expected answer



Using the law of conservation of momentum, the total momentum before the collision is equal to the total momentum after the collision.

$$(m_A u_A) + (m_B u_B) = (m_A v_A) + (m_B v_B)$$
$$\therefore (2 \times 5) + (3 \times 0) = (2 \times 0.5) + (3 \times v_B)$$
$$\therefore 10 = 1 + 3v_B$$
$$\therefore 3v_B = 9$$
$$\therefore v_B = 3.0\,\text{m s}^{-1}$$

Inelastic collisions (page 100)

Q1:

1. $m_1u_1 + m_2u_2 = (m_1 + m_2)\,v$
   $(0.22 \times 0.25) + 0.16u = (0.38 \times 0.2)$
   $0.055 + 0.16u = 0.076$
   $u = 0.13$ m s^{-1}
2. $E_k = \frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2$
   $E_k = 0.5 \times 0.22 \times 0.25^2 + 0.5 \times 0.16 \times 0.13^2$
   $E_k = 0.5 \times 0.22 \times 0.0625 + 0.5 \times 0.16 \times 0.0169$
   $E_k = 0.006875 + 0.001352$
   $E_k = 0.008227$ J
   Kinetic energy after :
   $E_k = {}^1/_2\,(m_1 + m_2)\,v^2$
   $E_k = 0.5 \times 0.38 \times 0.2^2$
   $E_k = 0.5 \times 0.38 \times 0.04$
   $E_k = 0.0076$ J
   Kinetic energy lost = 0.008227 - 0.0076 = 0.000627 J
3. It has been converted into other forms of energy, most probably heat and sound.
4. Less because total (initial) momentum is less but mass is constant and v = momentum / mass.

Ballistic pendulum (page 101)

Q2: We will use *m* for the mass of the bullet and *M* for the mass of the block.

$$\begin{aligned} PE &= (m+M)\,gh \\ \therefore PE &= (0.020+2.0)\times 9.8\times 0.42 \\ \therefore PE &= 8.3\,\text{J} \end{aligned}$$

Q3: The kinetic energy when the block and bullet are at their lowest height must be 8.3 J.

Q4:

$$\begin{aligned} E_k &= \tfrac{1}{2}(m+M)\,v^2 \\ \therefore v^2 &= \frac{2E_k}{(m+M)} \\ \therefore v^2 &= \frac{2\times 8.3}{2.02} \\ \therefore v^2 &= 8.218 \\ \therefore v &= 2.87\,\text{m s}^{-1} \end{aligned}$$

Q5: Before the collision, the block is stationary and the bullet is moving with velocity *u*. Immediately after the collision, the block and the bullet are both moving with the velocity we have just calculated.

$$\begin{aligned} mu &= (m+M)\,v \\ \therefore u &= \frac{(m+M)\,v}{m} \\ \therefore u &= \frac{2.02\times 2.87}{0.02} \\ \therefore u &= 290\,\text{m s}^{-1} \end{aligned}$$

Elastic collisions (page 104)

Q6:

1. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
   $4 \times 2 + 4 \times 0 = 4 \times 0 + 4\ v_2$
   $8 = 4\ v_2$
   $v_2 = 2.0\ \text{m s}^{-1}$
2. Kinetic energy before
   $E_k = \frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2$
   $E_k = 0.5\times 4\times 2^2 + 0.5\times 4\times 0^2$
   $E_k = 0.5\times 4\times 4\ +\ 0$
   $E_k = 8.0$ J
   Kinetic energy after

$E_k = \frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2$

$E_k = 0.5 \times 4 \times 0^2 + 0.5 \times 4 \times 02$

$E_k = 0 \ + \ 0.5 \times 4 \times 4$

$E_k = 8.0$ J

The kinetic energy before and after the collision is identical so the collision is elastic.

Quiz: Momentum (page 104)

Q7: d) 1.67 m s^{-1}

Q8: a) 3.8 m s^{-1}

Q9: e) total momentum is conserved but total kinetic energy is not.

Q10: c) 1.6 m s^{-1}

Q11: a) 0.60 m s^{-1}

Rocket (page 108)

Expected answer

1. $m_1u_1 + m_2u_2 = (m_1 + m_2)\ v$
   $2500 \times 0.5 + 1500\ u_2 = 4000 \times 0.2$
   $1250 + 1500\ u_2 = 800$
   $1500\ u_2 = 800 - 1250$
   $1500\ u_2 = -\ 450$
   $u_2 = {}^{-\,450} / {}_{1500}$
   $u_2 = -\ 0.3$ m s^{-1}

   The value is negative because the probe was moving in the opposite direction to the vehicle before the collision.

2.
   1. The space probe's engine.
   2. Momentum before = mv = 4000×0.2 = 800 kg m s^{-1}
      Momentum after = 0
      Change in momentum = final momentum - initial momentum = 0 - 800 = - 800 kg m s^{-1}
      The change in momentum is negative because it is to the left.
   3. The change in momentum of the gases must be the same in size but opposite in direction to the change in momentum of the space vehicle and probe. Change in momentum is + 800 kg m s^{-1}
      The change in momentum of the gases is positive because it is to the right.

Quiz: Explosions (page 109)

Q12: c) 2.4 m s^{-1}

Q13: e) 12 m s^{-1}

Q14: a)

$$\frac{v}{3}$$

Q15: d) 1.1 m s^{-1}

Q16: c) 7.2 m s^{-1}

Test your strength contest (page 112)

Expected answer

The block has gained potential energy 49 J when it is at the top of the pole, so its kinetic energy when it is at the foot of the pole must also be 49 J. From this, we can find the velocity of the block at the foot of the pole:

$$KE = \frac{1}{2}mv^2$$
$$\therefore v^2 = \frac{2KE}{m}$$
$$\therefore v^2 = \frac{2 \times 49}{1.0}$$
$$\therefore v = 9.9\,\text{m s}^{-2}$$

We are told that the impulse applied by the hammer acting downwards is equal to the impulse applied upwards to the block. Remember that impulse is equal to the change in momentum, so the change in momentum of the hammer is equal to the change in momentum of the block at the foot of the pole.

$$p_{hammer} = p_{block}$$
$$\therefore m_h u_h = m_b v_b$$
$$\therefore u_h = \frac{m_b v_b}{m_h}$$
$$\therefore u_h = \frac{1.0 \times 9.9}{1.5}$$
$$\therefore u_h = 6.6\,\text{m s}^{-1}$$

So the hammer has to be moving downwards with a minimum velocity of 6.6 m s^{-1} in order to make the block reach the top of the pole.

Car safety (page 113)

Q17: 12 m s^{-1} as it is travelling at the same speed as the car. There is no seat belt so the dummy continues to move in a straight line at a constant speed when the car stops.

Q18: Distance = (0.55 + 1.70) - 1.05 = 2.25 - 1.05 = 1.20 m

Q19: t = $^{s} / _{v}$ = $^{1.20} / _{12}$ = 0.1 s

Q20: Initial momentum of dummy = mu = 80 × 12 = 960 kg m s^{-1}
Final momentum of dummy = 0 kg m s^{-1}

Change in momentum = 0 - 960 = - 960 kg m s-^{1}. The change in momentum is negative because it is in the opposite direction to the initial velocity.

Q21: Change in momentum = F × t
$F = \frac{mv-mu}{t}$
F = $^{-960} / _{0.05}$
F = - 19200 N

The force is negative because it is a decelerating force.

Q22: The change in momentum of the dummy is the same as before but the air bag increases the time for the collision. As the change in momentum is equal to the time of contact times the force, if the time of contact increases the force must get smaller.

Pile driver (page 114)

Q23: $v^2 = u^2 + 2$ as
$v^2 = 0^2 + 2 \times 9.8 \times 2$
$v^2 = 39.2$
v = 6.26 m s^{-1}

Q24: Initial momentum = mu = 15 × 6.26 = 93.9 kg m s^{-1}

Final momentum = 0 kg m s^{-1}

Change momentum = *mv - mu*

Change in momentum = 0 - 93.9 = - 93.9 kg m s^{-1}

The change in momentum is negative because it is in the opposite direction to the direction the mass is travelling.

Q25: Change in momentum = force × time so we can calculate the average force of the pipe on the mass.
F = $^{Dp} / _{t}$ = $^{-93.9} / _{0.02}$ = - 4700 N

This force is the force of the pipe on the mass. According to Newton's third law the mass must exert an equal but opposite force on the pipe.

The force on the pipe is therefore + 4700 N.

Since the force is positive, the force is in the downwards direction. . .

Q26: The size of the force will be less.

The change in momentum of the mass will be the same.

The soft material will increase the time of contact between the mass and the pipe.

$F = \frac{mv-mu}{t}$

As t increases then F must decrease.

Quiz: Impulse (page 117)

Q27: b) 3200 kg m s^{-1}

Q28: c) force = rate of change of momentum.

Q29: e) 60 kg m s^{-1}

Q30: a) momentum.

Q31: d) 4800 N

End of topic 3 test (page 120)

Q32:

138 m s^{-1}

Q33: 0.0375 J

Q34:

1. -3.0 m s^{-1}
2. +5.0 m s^{-1}

Q35: 11 kg m s^{-1}

Q36: 1.26 m

Q37: 1.4 m s^{-1}

Q38: 5.8 m s^{-1}

Q39: 257 m s^{-1}

Q40: 290 N

Q41: 0.459 kg m s^{-1}

Q42:

1. -30 kg m s^{-1}
2. -16 m s^{-1}

Q43:

1. 6 m s^{-1}
2. 12 kg m s^{-1}

Q44:

1. 14 N s
2. 17.5 m s^{-1}

Topic 4: Gravitation

Projectile motion (page 134)

Q1: First we need to find the time it takes to fall the 5 m.

$s = ut + {}^{1}/_{2}\ at^2$

$u = 0$
$s = {}^{1}/_{2}\ at^2$

$t^2 = {}^{2s}/_{a}$
$t^2 = 2 \times 5 / 9.8$
$t^2 = {}^{10}/_{9.8}$
$t^2 = 1.02$

$t = 1.01$ s

Now find the horizontal distance the projectile travels

$s = vt$
$s = 10 \times$x 1.01
$s = 10.1$ m

The ball will land 10.1 m from the wall.

Q2: First we need to find the horizontal and vertical components of the velocity.

Velocity horizontal component = v cos q

$v_h = 10 \cos 30 = 8.7$ m s^{-1}

Velocity vertical component = v sin q

$v_v = 10 \sin 30 = 5.0$ m s^{-1}

Again we have to find the time of flight for the ball.

Let up be the positive direction.

$s = ut + {}^{1}/_{2}\ at^2$

u is the initial vertical velocity.

$-5 = 5t + {}^{1}/_{2} \times -9.8 \times t^2$
$-5 = 5t - 4.9t^2$

Solving for t we get 1.64 s.

To find the horizontal distance we multiply the horizontal component by the time.

$s = vt$
$s = 8.7 \times 1.64$
$s = 14.3$ m

The ball will land 14.3 m from the wall.

We can use this method to find the range of a projectile if we know the initial speed, the angle to the horizontal and the height.

First we find the components of the velocity.

Second we find the time of flight.

Finally we multiply the horizontal component by the time of flight to get the range.

End of topic 4 test (page 138)

Q3:

1. 0.39 s
2. 4.35 m s^{-1}

Q4: 5970 m

Q5:

1. 5.73 m
2. 6.09 m

Q6:

1. s = 2.5 m
2. s = 2.2 m

Topic 5: Gravity and mass

Quiz: Mass, weight and gravitational field strength (page 142)

Q1: a) *W* = *m* x *g*

Q2: e) 245 N

Q3: c) Only the weight of the object is affected by the gravitational field strength.

Q4: d) 640 N

Q5: e) decreases to zero.

Acceleration due to gravity (page 147)

Q6:

time /s	Earth	Jupiter	Mars	Mercury	Neptune	Saturn	Venus
0	100	100	100	100	100	100	100
1	95.1	87	98	98	94	94.5	95.5
2	80.4	48	92	92	76	78	82
3	55.9	0	82	82	46	50.5	59.5
4	21.6	0	68	68	4	12	28
5	0	0	50	50	0	0	0
6	0	0	28	28	0	0	0
7	0	0	2	2	0	0	0
8	0	0	0	0	0	0	0

Quiz: Gravitational force (page 148)

Q7: d) 1.0×10^{-10} N

Q8: a) $F_S = F_E$

Q9: d) 8.10 N

Q10: b) Its mass remains constant but its weight decreases.

Q11: e) 8.87 m s^{-2}

End of topic 5 test (page 150)

Q12: 1.46×10^{20} N

Q13: 2.3×10^{-10} N

Q14: 74.2 N

Q15: 9.42 m s^{-2}

Q16: 2.8 N

Q17: 5.17 N kg^{-1}

Q18: 0.428 N kg^{-1}

Q19: 7.82×10^{-3} N kg^{-1}

Topic 6: Special relativity

End of topic 6 test (page 164)

Q1: 192 seconds

Q2: 1000000 m s^{-1}

Q3: 176 metres

Topic 7: The expanding universe

Doppler effect (page 169)

Expected answer

(1) Use the formula

$$f' = f\frac{v}{v - v_s}$$

Putting in the values of f = 400 Hz, v = 340 m s^{-1}, v_s = 15 m s^{-1}

$$f' = 400 \times \frac{340}{340 - 15}$$
$$\therefore f' = 400 \times \frac{340}{325}$$
$$\therefore f' = 418 \text{ Hz}$$

(2) Now, use the formula

$$f' = f\frac{v}{v + v_s}$$

$$f' = 400 \times \frac{340}{340 + 15}$$
$$\therefore f' = 400 \times \frac{340}{355}$$
$$\therefore f' = 383 \text{ Hz}$$

Quiz: Doppler effect (page 169)

Q1: b) frequency.

Q2: c) 495 Hz

Q3: a) higher when the man is walking towards her.

Q4: b) 10.6 m s^{-1}

End of topic 7 test (page 175)

Q5:

1. 0.001
2. 526 nm

Q6:

1. 1.025×10^6 m s $^{-1}$
2. 582 nm

Q7: 498 Hz

Q8: 17.6 m s^{-1}

Q9:

1. 26.5 m s $^{-1}$
2. 497 Hz

Topic 8: Hubble's Law

End of topic 8 test (page 184)

Q1: 2.14×10^{-18}

Change units of answer to . . . s^{-1}.

Q2: 18.1 km s^{-1} / million light years

Topic 9: Expansion of the universe

End of topic 9 test (page 190)

Q1:

a) Ordinary or visible mass/matter

b) Dark matter

Q2: b) Red shift of light

Q3: b) mass of the universe.

Q4: c) dark energy.

Topic 10: The Big Bang theory

End of topic 10 test (page 198)

Q1: Rigel

Q2: Blue giants

Q3: Graph 1

Topic 11: End of unit tests

Open ended and skill based questions (page 200)

Q1: *The following points could be included in your answer:*

The passenger is not thrown forward.

Both are moving at the same starting velocity, 30 m s^{-1} and both end at 0 m s^{-1}.

The passenger continues with a constant velocity of 30 m s^{-1} because there is no unbalanced force acting on them.
When they hit something an unbalanced force is applied to them and they stop.
If they hit something "hard" they will stop is a short time.
Stopping in a short time means the acceleration or deceleration is large. When the stopping time is short then the applied force is large.

The driver does stop with the car.
The seat belt stretched slightly.
The seatbelt stops the driver over a longer time than the stopping time for the passenger.
A smaller force stops the driver.

The following equations could be included in your answer:

$$F = m \times a$$
$$a = \frac{v - u}{t}$$
$$F \times t = mv - mu$$

Your answer must be structured so that it is logical when read.

Q2: *Estimates:*

Mass of sprinter = 80 kg
Initial velocity = 8 m s^{-1}
Final velocity = 0 m s^{-1}
Stopping time = 0.4 s

Calculations:

$$F \times t = mv - mu$$
$$F \times 0.4 = 80 \times 0 - 80 \times 8$$
$$F = -1600N$$

Watch you must have v = 0 m s^{-1}

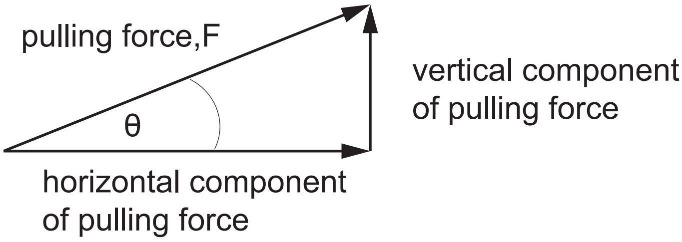
Q3: *The following points could be included in your answer:*

Force of friction opposes motion of box.
When the force is applied horizontally, the force of friction is greater than pulling force

When force is applied at an angle, there is a component of the applied force in the vertical direction.
When force is applied at an angle, there is a component of the applied force in the horizontal direction.
The vertical component lifts part of the box off the floor.
This reduces force of friction.
The horizontal component of the applied force is greater than the new force of friction.

There is an unbalanced force acting horizontally.
The box accelerates across floor.

The following equations could be included in your answer:



Horizontal component of pulling force = F cosΘ
Vertical component of pulling force = F sinΘ
F_{un} = m x a

Your answer must be structured so that it is logical when read.

Q4: The commentator's description is in three parts so structure your answer so that each of these three parts is covered.

The following points could be included in your answer:

The ball acts as a projectile.
Need to consider horizontal and vertical components of velocity.
Normally expect horizontal component of velocity to remain constant but here it decreases due to the force of the wind.

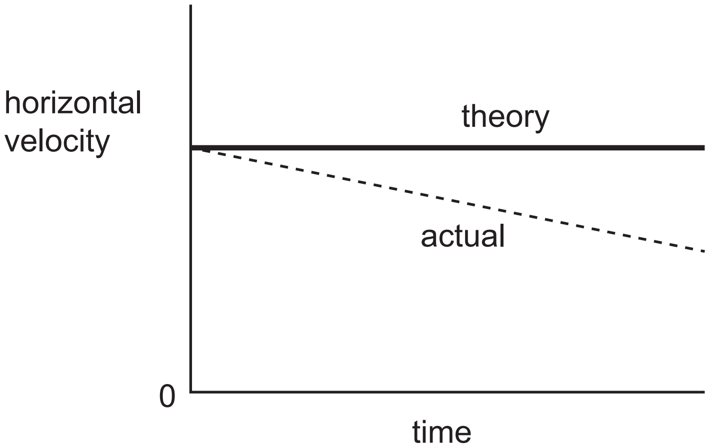
After the ball is hit, it rises and as it rises its vertical component of velocity will decrease due to the force of gravity.
The ball accelerates vertically downwards at 9.8 m s^{-2} ignoring force of air resistance.
The ball does seem to soar into the air because it is initially moving fast but its vertical component of velocity decreases.

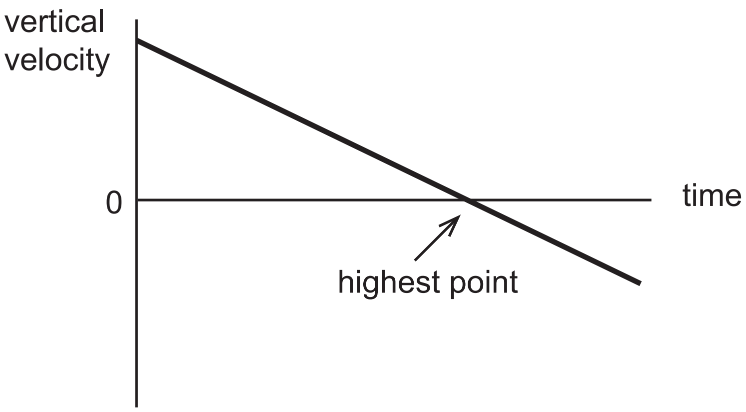
At the highest point the ball, as it rises into the air, will stop vertically for an instant but it will still have a horizontal velocity.
The ball therefore never stops.

All through the flight the horizontal component of velocity decreases hence it falls more steeply than it climbed.

The ball does not get slower as it falls to the ground.
It accelerates downwards so the vertical component of the velocity will increase.
The horizontal component of the velocity will decrease.
The resultant velocity of the ball will be more vertical than when it was hit.

The following graphs could be included in your answer:





Your answer must be structured so that it is logical when read.

Q5: *Step 1 find gradient:*

There are many different ways of writing the first line of the gradient calculation.

$$\begin{aligned} \text{gradient} &= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}} \\ &= \frac{\Delta \text{ pressure}}{\Delta \text{ depth}} \\ &= \frac{2000 - 0}{0.25 - 0} \\ &= 8000 \ \left(\text{Pa m}^{-1}\right) \end{aligned}$$

Step 2 find density of liquid:

$$\begin{aligned} P &= pgh \\ \frac{P}{h} &= pg \\ \text{gradient} = \frac{P}{h} &= pg \\ 8000 &= pg \\ 8000 &= p \times 9.8 \\ p &= 820 \text{ kg m}^{-3} \end{aligned}$$

Note: you must use the gradient of the best line and not individual data points when calculating the gradient.

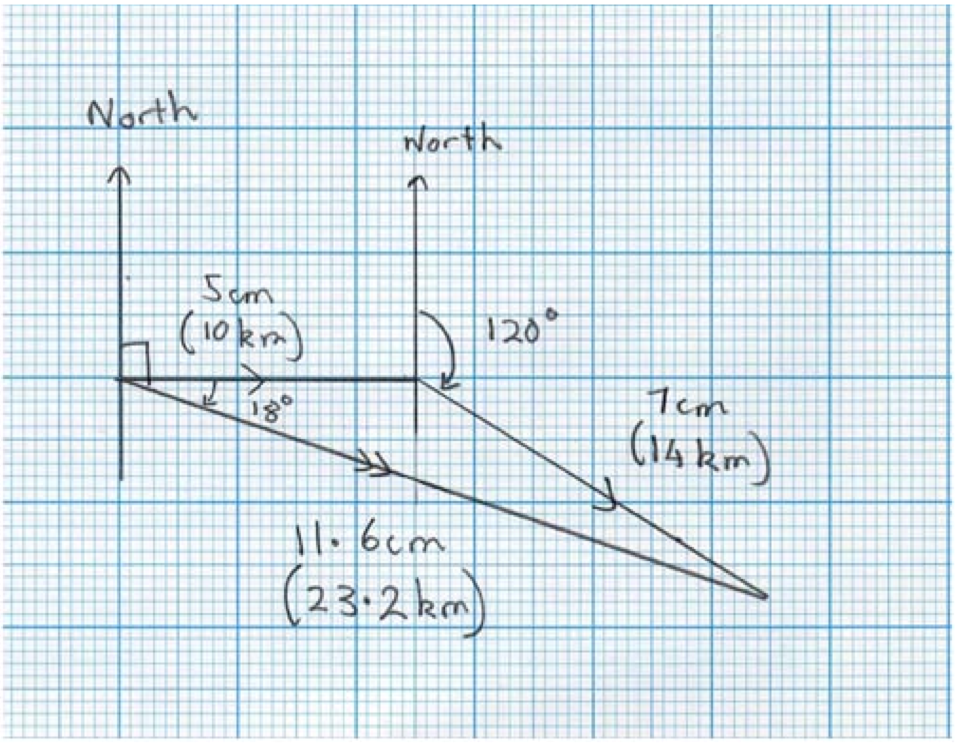
Course style questions (page 202)

Q6:

1. A scalar quantity has only magnitude (size).
   A vector quantity has both magnitude (size) and direction.

(1)

2. a)



23.2 km at bearing of 108

(3)

b)
s = 23.2 km = 23200 at 108
t = 20 + 35 = 55 minutes = 3300 s

$$v = \frac{s}{t} = \frac{23200}{3300} = 7.0 \text{ ms}^{-1} \text{ at } 108$$

(3)

Must remember to quote bearing as well as 7.0 m s^{-2}.

3. a)

$$F = m \times a$$
$$70 = 120 \times a$$
$$a = 0.58 \text{ ms}^{-2}$$

(2)

Must start with equation, otherwise zero marks.

b) $v = u + at$
$14 = u + 0.58 \times 20$
$u = 2.4\ m\ s^{-1}$

(3)

Must substitute value for v and find u, not substitute u and find v.

Marks (12)

Q7:

1. a) $u_{horiz} = u \cos \Theta$
   $u_{horiz} = 25 \cos 37$
   $u_{horiz} = 20\ m\ s^{-1}$

   (1)

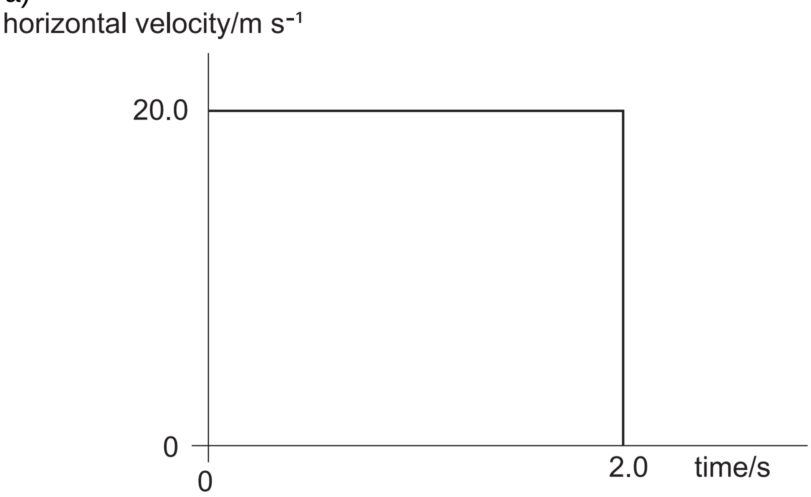
   b) $u_{vert} = u \sin \Theta$
   $u_{vert} = 25 \sin 37$
   $u_{vert} = 15\ m\ s^{-1}$

   (1)

   c) Find maximum height of ball.
   $v^2 = u^2 + 2as$
   $0^2 = 15^2 + 2 \times -9.8 \times s$
   $s = 11.5\ m$
   Height of hilltop = 11.5 - 4 = 7.5 m

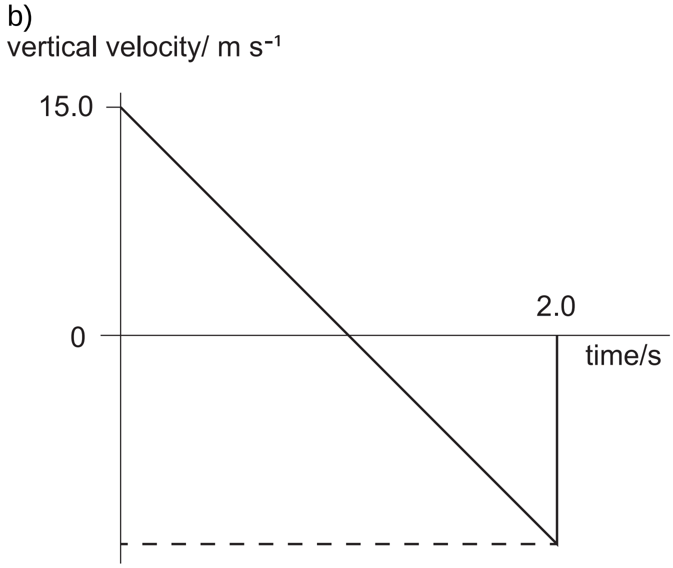
   (4)

2. a)



(2)

b)



(2)

3. Any value less than the 7.5 m, calculated in 1.c).
   Justification; max height will be less since the frictional forces will reduce velocity of ball **more than before**.
   Or ball will reach highest point earlier in flight
   Or time in air will be less
   Or maximum height reached will be less

(2)

Marks (12)

Q8: a) Impulse = area under graph

$$= \frac{1}{2} \times base \times height$$
$$= \frac{1}{2} \times 0.6 \times -8.2$$
$$= -2.46 \text{ Ns}$$

(3)

The negative sign is due to the impulse being in a direction opposite to the motion which is downwards.

---

b) Direction of impulse on brick is upwards.

(1)

c) impulse = mv - mu
-2.46 = m x 0 - m x 7.6
m = 0.32kg

(3)

It is v, the final velocity that is zero.

d) Since the rubber is harder the force exerted will be greater.

By $F \times t = mv - mu$ where m, v and u are all constant $\Rightarrow$ increasing F will decrease stopping time t.

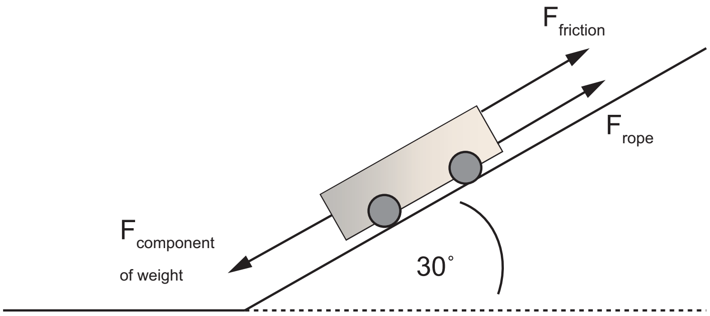
(2)

In answering "explain/justify" questions it is useful to state the relevant formula and ensure that each letter in the formula is considered.

Marks (9)

Q9:

a)



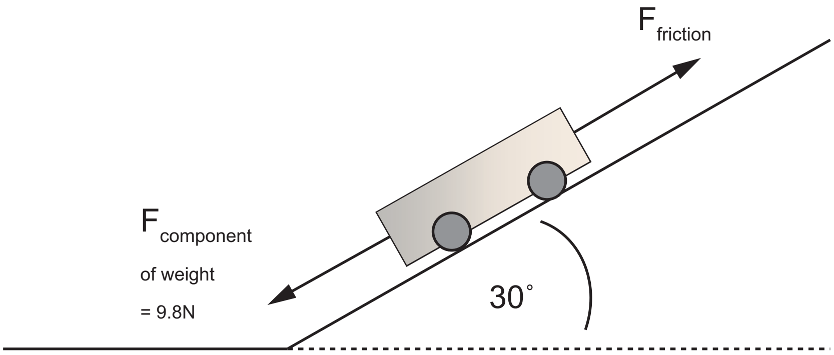
(1)

Must have arrows on lines and names beside each arrow.

b) $F_{comp\ of\ weight} = mg \sin \Theta$
= 2 x 9.8 x sin 30
= 9.8N

(3)

c) $F_{un} = ma$
= 2 x 1.5
= 3.0N



Force of friction acting against the motion of trolley = 9.8 - 3 = 6.8 N

(4)

d) When moving up slope both the component of weight and force of friction are acting in the same direction; down the slope.
⇒ greater unbalanced force (9.8 + 6.8 = 16.6 N down slope).
⇒ greater acceleration.

(2)

Marks (10)

Q10:

1. a)

$$t = \frac{t}{\sqrt{1-(\frac{v}{c})^2}}$$
$$4.3 = \frac{t}{\sqrt{1-(\frac{0.95c}{c})^2}}$$
$$4.3 = \frac{t}{0.31}$$

t = 1.3 years

(3)

b)

$$l' = l\sqrt{1-(\frac{v}{c})^2}$$
$$l' = l\sqrt{1-(\frac{0.95c}{c})^2}$$
$$l' = 4.3 \times 0.31$$

l' = 1.3 light-years

(3)

c) The diameter measured by the stationary observer on Earth and the astronaut will be the same as length contraction only occurs in the direction of travel.

(1)

2. a) Light from all galaxies is red shifted.
The further away a galaxy is, the greater the red shift and therefore the faster the galaxy is moving.

(2)

b) You should know:

$$v = H_o \times d \Rightarrow H_0 = \frac{v}{d}$$

Must find the gradient as an estimate of Hubble's constant.
You must find the gradient of the line.
You must not use data points on the graph.

$$gradient = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$
$$gradient = \frac{(124000 - 0)}{(5 \times 10^{22} - 0)}$$

gradient = 2.5 x 10^{-18} s^{-1}

(3)

Marks (12)

End of unit 1 test (page 207)

Q11: b) displacement when the acceleration is zero

Q12: 31 m

Q13: 21 m

Q14:

1. 1.6 s
2. 4.2 s
3. 38.5 J
4. 20.7 m

Q15:

1. 13 N
2. 0.13 m s^{-2}
3. 1.6 m

Q16: e) 1.5 d

Q17:

1. 1.5 m s^{-1}
2. 190 J
3. 11 m

Q18: e) (5.0 cos 55° + 14.7)

Q19:

1. 8.2 m s^{-1}
2. 17 m

Q20: 37.2 N

Q21: c) 12 N kg^{-1}

Q22: 9.96×10^{3}

Q23: 2.71×10^{20} N

Q24: 8.5×10^{10} N

Q25: 63 N

Q26: 3.81 N

Q27: The apparent frequency *f* ' is given by the equation

$$f_o = f_s \times \frac{v}{v - v_s}$$

where *v* is the speed of sound.

Q28: 30.3 Hz

Q29: 24 Hz

Q30:

1. 9.8 m s $^{-2}$
2. 30.0

Q31:

1. $E_k = {}^1/_2\ m \times v^2$
   $E_k = {}^1/_2 \times 1000 \times 20^2$
   $E_k = {}^1/_2 \times 1000 \times 400$
   $E_k = 200000$
   $E_k = 2.0 \times 10^5$ J
2. $P = {}^E/_t$
   $P = {}^{200000}/_{40}$
   P = 5000 W

3. $F = {}^W/_d$
   $F = {}^{200000}/_{25}$
   F = 8000 N

Q32:

1. p = mv
   $p = 0.5 \times 4$
   p = 2.0 kg m s^{-1}.
2. p = - 2.0 kg m s^{-1}.
3. $v = {}^p/_m$
   $v = {}^{-2.0}/_{20}$
   v = - 0.1 m s^{-1}.

Q33:

1. Impulse = area under the graph
   Impulse = ${}^1/_2 \times 0.2 \times 8$
   Impulse = 0.8 Ns
2. Change in momentum = impulse
   Change in momentum = 0.8 kg m s^{-1}
3. $v = {}^p/_m$
   $v = {}^{0.8}/_{0.02}$
   v = 40 m s^{-1}

Q34: $l' = 20\sqrt{(1 - (0.5)^2)}$
$l' = 20\sqrt{0.75}$
l' = 17.32 m

Appendix A: Units, prefixes and scientific notation

Quiz questions (page 221)

Q1: c) 1.3

Q2: d) 39 J

Q3: c) 40.9 J